INTERNATIONAL JOURNAL OF GEOMAGNETISM AND AERONOMY
VOL. 7, GI1005, doi:10.1029/2005GI000123, 2007

Efficiency of the Earth–ionosphere waveguide excitation by ELF sources located in an anisotropic ionosphere

S. T. Rybachek[1] and M. Ju. Ponomariev[1]

Received 5 April 2005; revised 4 July 2006; accepted 24 October 2006; published 31 March 2007.

[1] This paper is devoted to the problem of excitation of the Earth–ionosphere waveguide by the sources located in an anisotropic ionosphere. The ELF range where a single-mode representation of the field can be applied is considered. The generalized theorem of reciprocity makes it possible to find the relation between the field components excited in the waveguide by electric and magnetic dipoles of various orientation. At the location of the emitters in the field of applicability of the quasi-longitudinal approximation, this relation is described by simple analytical expressions. Some results of calculations characterizing the efficiency of waveguide excitation by horizontal and radial dipoles and applicability of approximate formulae are presented. The dependencies of the efficiency parameters on the frequency, latitude, height of the emitter location, and propagation conditions are analyzed. *INDEX TERMS:* 6964 Radio Science: Radio wave propagation; 2403 Ionosphere: Active experiments; 2471 Ionosphere: Plasma waves and instabilities; *KEYWORDS:* Earth–ionosphere waveguide; ELF radiation; lower ionosphere.

Citation: Rybachek, S. T. and M. Ju. Ponomariev (2007), Efficiency of the Earth–ionosphere waveguide excitation by ELF sources located in an anisotropic ionosphere, *Int. J. Geomagn. Aeron., 7,* GI1005, doi:10.1029/2005GI000123.

1. Introduction

[2] The properties of electromagnetic fields excited in the terrestrial waveguide by low-frequency elementary emitters located in the ionosphere were studied in a series of papers. Various models of the geomagnetic field and terrestrial regular waveguide with the properties depending only on the coordinate orthogonal to the boundaries [see, e.g., *Einaudi and Wait*, 1971a, 1971b; *Galejs*, 1971a; *Pappert*, 1973; *Rybachek*, 1985, 1995] and irregular waveguide with the properties changing additionally in the longitudinal (tangential) to the boundaries direction [*Galejs*, 1971b; *Rybachek et al.*, 1997a, 1997b] were used.

[3] In our papers we used the model of a spherical anisotropic waveguide with an arbitrary orientation of the geomagnetic field vector $\mathbf{H}_0$. Point electric and magnetic dipoles of arbitrary orientation were considered as the field sources. The cold plasma approximation was used to describe the ionospheric properties. Nonlinear effects were not taken into account.

[4] The solution of the problem of the terrestrial waveguide excitation by ionospheric emitters is found using the generalized theorems of reciprocity for magnetoactive media [*Ginzburg*, 1970] and is reduced to determination of ionospheric fields created by the dipoles located in the cavity of the waveguide. The latter problem, in its turn, leads to a series of waveguide problems (the source and observation point are located in the waveguide cavity) and to the problem of integration of the Maxwell's equations for anisotropic media. The solution of waveguide problems is obtained by the normal waves method for a regular waveguide [*Krasnushkin*, 1962] and by the cross sections method for an irregular waveguide [*Katsenelenbaum*, 1961]. The problem of determination of the fields in the anisotropic ionosphere using the asymptotic separation of variables in the Maxwell's equations is reduced to an integration of the system of differential equations, coinciding in the form with the system describing propagation of plane waves through plane-stratified media [*Budden*, 1961].

[5] This paper is dedicated to studies of the efficiency of excitation of the Earth–ionosphere waveguide by horizontal and vertical electric and magnetic ionospheric dipoles in the ELF (extremely low frequencies) range where the single-mode representation of the field is mainly applicable.

[1]Institute of Radio Physics, St. Petersburg State University, St. Petersburg, Russia

1524–4423/07/2005GI000123$18.00

2. Formulation and Construction of the Solution

[6] The Earth–ionosphere waveguide is modeled by a spherical cavity characterized by the properties of the free space. In the geocentric spherical coordinate system r, θ, φ, the axis $\theta = 0$ goes through the emitter located in the ionosphere in point 1 with coordinates $r = b$ and $\theta = 0$. The ground $(r \leq a)$ and the ionosphere $(r \geq d)$ are considered as inhomogeneous media depending on r and θ. Point electric and magnetic dipoles oriented along the unit vectors $\mathbf{e}_\xi$ $(\xi = r, \theta, \varphi)$ and oscillating according to $\exp(-i\omega t)$ are used as the field sources (ω is the angular frequency). The IS system of units is used. Our aim is to find the electric $\mathbf{E}$ and magnetic $\mathbf{H}$ fields in a point 2 with coordinates r, θ located in the waveguide cavity.

[7] The generalized reciprocity theorem for magnetoactive medium makes it possible to find components of the fields excited in the waveguide cavity by ionospheric dipoles using components of the fields excited in the ionosphere by the dipoles located in the waveguide cavity:

$$E_\xi^{ie\zeta}(1,\, 2,\, \mathbf{H}_0) = E_\zeta^{e\xi}(2,\, 1,\, -\mathbf{H}_0)$$

$$\mathcal{H}_\xi^{ie\zeta}(1,\, 2,\, \mathbf{H}_0) = -gE_\zeta^{m\xi}(2,\, 1,\, -\mathbf{H}_0)$$

$$gE_\xi^{im\zeta}(1,\, 2,\, \mathbf{H}_0) = -\mathcal{H}_\zeta^{e\xi}(2,\, 1,\, -\mathbf{H}_0)$$

$$\mathcal{H}_\xi^{im\zeta}(1,\, 2,\, \mathbf{H}_0) = \mathcal{H}_\zeta^{m\xi}(2,\, 1,\, -\mathbf{H}_0) \qquad (1)$$

Here $\mathcal{H}_p \equiv z_0 H_p$, $p = \xi, \zeta$, where $z_0 = 120\pi$ [Ohm] is the characteristic impedance of free space. The indices e and m at the field components refer to the electric and magnetic dipoles, respectively. For example, $E_\xi^{ie\zeta}(1,\, 2,\, \mathbf{H}_0)$ and $\mathcal{H}_\xi^{ie\zeta}(1,\, 2,\, \mathbf{H}_0)$ are the ξ components of the fields excited in the waveguide cavity in point 2 by the electric dipole with the dipole moment $\mathbf{P}_e^{i\zeta}$ located in the ionosphere in point 1 and oriented along the unit vector $\mathbf{e}_\zeta$; $E_\zeta^{e\xi}(2,\, 1,\, -\mathbf{H}_0)$ and $\mathcal{H}_\zeta^{e\xi}(2,\, 1,\, -\mathbf{H}_0)$ are the ζ components of the electric and magnetic fields, respectively, excited in point 1 in the ionosphere at the geomagnetic field $-\mathbf{H}_0$ by an auxiliary electrical dipole with the moment $\mathbf{P}_e^\xi$ located in point 2 and oriented along the unit vector $\mathbf{e}_\xi$ (ξ, ζ= r, θ, φ). In expressions (1) it is assumed that

$$P_e^{i\xi} = P_e^\xi \equiv P_e$$

$$P_m^{i\xi} = P_m^\xi \equiv P_m$$

$$g \equiv z_0 P_e P_m^{-1} \qquad (2)$$

3. Relations of Field Components Excited in the Ionosphere by Dipoles Located in the Waveguide

[8] The solutions of waveguide problems for the anisotropic, irregular along the θ coordinate waveguide can be carried out approximating the irregular part by a finite number of regular parts [*Rybachek et al.*, 1997a]. The waveguide problem for the regular part [*Rybachek*, 1995] is reduced to a solution of the Maxwell's equations in the waveguide cavity for electric dipoles

$$\mathrm{rot}\,\mathbf{E} = ik\mathcal{H}$$

$$\mathrm{rot}\,\mathcal{H} = -ik(\mathbf{E} + \mathbf{p}_e/\varepsilon_0) \qquad (3)$$

or for magnetic dipoles

$$\mathrm{rot}\,\mathbf{E} = ik(\boldsymbol{\mathcal{H}} + z_0\mathbf{p}_m/\mu_0)$$

$$\mathrm{rot}\,\boldsymbol{\mathcal{H}} = -ik\mathbf{E} \qquad (4)$$

In (3) and (4), k is the wave number of the free space and $\mathbf{p}_e$ and $\mathbf{p}_m$ are the volume densities of the dipole moments of the electric and magnetic dipoles, respectively. The boundary conditions at both boundaries of the waveguide have the following form:

$$\begin{pmatrix} E_\theta \\ E_\varphi \end{pmatrix} = \widehat{\delta}\begin{pmatrix} \mathcal{H}_\theta \\ \mathcal{H}_\varphi \end{pmatrix} \qquad r = a$$

$$\begin{pmatrix} \mathcal{H}_\theta \\ \mathcal{H}_\varphi \end{pmatrix} = \widehat{a}\begin{pmatrix} E_\theta \\ E_\varphi \end{pmatrix} \qquad r = d \qquad (5)$$

Besides, the conditions of finiteness of the fields $\mathbf{E}$ and $\boldsymbol{\mathcal{H}}$ under $\theta = 0, \pi$ should be fulfilled. In expressions (5), $\widehat{a}$ is a matrix of the ionospheric admittance. The matrix $\widehat{\delta}$ have the following form:

$$\widehat{\delta} = \begin{pmatrix} 0 & -\delta \\ \delta & 0 \end{pmatrix} \qquad (6)$$

where δ is the relative surface impedance of the ground which is taken the same for both polarizations.

[9] According to the principle of polarization duality, the general solution of the Maxwell's equations is given by a superposition of two fundamental solutions. The transverse magnetic and transverse electric fields are described by the electric $\boldsymbol{\Pi}_e = \Pi_e\mathbf{e}_r$ and magnetic $\boldsymbol{\Pi}_m = \Pi_m\mathbf{e}_r$ Hertz vectors, respectively, directed along the separation coordinate r. In the waveguide cavity outside the sources the electric and magnetic field vectors can be derived as

$$\boldsymbol{\mathcal{H}}_e = -ik\,\mathrm{rot}\,\boldsymbol{\Pi}_e$$

$$\mathbf{E}_e = \mathrm{rot}\,\mathrm{rot}\,\boldsymbol{\Pi}_e$$

$$\mathbf{E}_m = ik\,\mathrm{rot}\,\boldsymbol{\Pi}_m$$

$$\boldsymbol{\mathcal{H}}_m = \mathrm{rot}\,\mathrm{rot}\,\boldsymbol{\Pi}_m \qquad (7)$$

The solution of the problem is obtained by the normal waves method. The dependence of the potentials on the coordinates r and θ with allowance for asymptotic presentations of the Legendre functions $P_\nu(-\cos\theta)$ applicable at $|\nu|\theta \gg 1$, $|\nu|(\pi-\theta) \gg 1$ [*Erdelyi*, 1953] for each normal wave is described by the following functions [*Makarov et al.*, 1994; *Rybachek et al.*, 1997a]:

$$\Pi_e \sim R_\nu^{(e)}(kr)\exp(i\nu\theta)$$
$$\Pi_m \sim R_\nu^{(m)}(kr)\exp(i\nu\theta) \qquad (8)$$

Here ν is the eigenvalue and $R_\nu^{(e)}(kr)$ and $R_\nu^{(m)}(kr)$ are the eigenfunctions of the radial operator of the problem satisfying the differential equations

$$\frac{d^2}{dr^2}\begin{pmatrix} R_\nu^{(e)}(kr) \\ R_\nu^{(m)}(kr)\end{pmatrix} = \frac{\nu(\nu+1)}{r^2}\begin{pmatrix} R_\nu^{(e)}(kr) \\ R_\nu^{(m)}(kr)\end{pmatrix}$$

and the boundary conditions at the surface of the ground at $r = a$ following from (5) and (6)

$$R_\nu^{(e)\prime}(kr) = -i\delta R_\nu^{(e)}(kr)$$
$$R_\nu^{(m)\prime}(kr) = -\frac{i}{\delta}R_\nu^{(m)}(kr) \qquad (9)$$

where the prime denotes a derivative with respect to the argument and the impedance δ is taken independent on the spectral parameter. The boundary conditions at the upper ionospheric boundary determined by the elements of the admittance $\widehat{a}(\nu)$ make it possible to obtain the characteristic equation for the eigenvalues.

[10] According to (7), one can present the tangential components of the fields in the waveguide cavity outside the sources in the single-mode approximation, omitting the e and m indices at field components, in the following way:

$$\begin{pmatrix} E_\theta \\ \mathcal{H}_\theta\end{pmatrix} = \frac{1}{r}\frac{\partial^2}{\partial r\partial\theta}\begin{pmatrix}\Pi_e \\ \Pi_m\end{pmatrix}$$
$$\begin{pmatrix} E_\varphi \\ \mathcal{H}_\varphi\end{pmatrix} = \frac{ik}{r}\frac{\partial}{\partial\theta}\begin{pmatrix}-\Pi_m \\ \Pi_e\end{pmatrix} \qquad (10)$$

The radial components can be expressed via potentials using (7) and the differential equations determining the transverse (radial) and longitudinal operators of the problem:

$$\begin{pmatrix} E_r \\ \mathcal{H}_r\end{pmatrix} = \frac{\nu(\nu+1)}{r^2}\begin{pmatrix}\Pi_e \\ \Pi_m\end{pmatrix} \qquad (11)$$

Using expressions (8), (10), and (11) and taking $|\nu| \gg 1$, we find the ratios of the field components in the waveguide cavity

$$\frac{E_\theta}{E_r} = \frac{ikrR_\nu^{(e)\prime}(kr)}{\nu R_\nu^{(e)}(kr)}$$

$$\frac{\mathcal{H}_r}{\mathcal{H}_\theta} = \frac{\nu R_\nu^{(m)}(kr)}{ikrR_\nu^{(m)\prime}(kr)}$$

$$\frac{E_\theta}{\mathcal{H}_\varphi} = \frac{R_\nu^{(e)\prime}(kr)}{iR_\nu^{(e)}(kr)}$$

$$\frac{E_\varphi}{\mathcal{H}_\theta} = -\frac{iR_\nu^{(m)}(kr)}{R_\nu^{(m)\prime}(kr)} \qquad (12)$$

If the point of observation is located on the surface of the ground, then, because of the boundary conditions (9), the ratios (12) take the following form:

$$\frac{E_\theta}{E_r} = \frac{ka\delta}{\nu}$$

$$\frac{\mathcal{H}_r}{\mathcal{H}_\theta} = \frac{\nu\delta}{ka}$$

$$\frac{E_\theta}{\mathcal{H}_\varphi} = -\delta$$

$$\frac{E_\varphi}{\mathcal{H}_\theta} = \delta \qquad (13)$$

Ratios (12) and (13) are valid for any height of the emitter located in the waveguide cavity.

[11] The relations between the components of the fields excited in the ionosphere in point 2 with coordinates (r, θ) by various emitters located in the waveguide cavity in point 1 with coordinates $(b, 0)$ $(b \le d)$ can be obtained using expressions (1):
for the electric dipoles (oriented along the unit vector $\mathbf{e}_\theta$ and radial):

$$\frac{E_\zeta^{e\theta}(1,\,2,\,\mathbf{H}_0)}{E_\zeta^{er}(1,\,2,\,\mathbf{H}_0)} = \frac{E_\theta^{ie\zeta}(2,\,1,\,-\mathbf{H}_0)}{E_r^{ie\zeta}(2,\,1,\,-\mathbf{H}_0)}$$
$$\frac{\mathcal{H}_\zeta^{e\theta}(1,\,2,\,\mathbf{H}_0)}{\mathcal{H}_\zeta^{er}(1,\,2,\,\mathbf{H}_0)} = \frac{E_\theta^{im\zeta}(2,\,1,\,-\mathbf{H}_0)}{E_r^{im\zeta}(2,\,1,\,-\mathbf{H}_0)} \qquad (14)$$

for the magnetic dipoles (oriented along the unit vector $\mathbf{e}_\theta$ and radial):

$$\frac{E_\zeta^{mr}(1,\,2,\,\mathbf{H}_0)}{E_\zeta^{m\theta}(1,\,2,\,\mathbf{H}_0)} = \frac{\mathcal{H}_r^{ie\zeta}(2,\,1,\,-\mathbf{H}_0)}{\mathcal{H}_\theta^{ie\zeta}(2,\,1,\,-\mathbf{H}_0)}$$
$$\frac{\mathcal{H}_\zeta^{mr}(1,\,2,\,\mathbf{H}_0)}{\mathcal{H}_\zeta^{m\theta}(1,\,2,\,\mathbf{H}_0)} = \frac{\mathcal{H}_r^{im\zeta}(2,\,1,\,-\mathbf{H}_0)}{\mathcal{H}_\theta^{im\zeta}(2,\,1,\,-\mathbf{H}_0)} \qquad (15)$$

for the horizontal dipoles (electric oriented along the unit vector $\mathbf{e}_\theta$ and magnetic oriented along the unit vector $\mathbf{e}_\varphi$):

$$\frac{E_\zeta^{e\theta}(1,\,2,\,\mathbf{H}_0)}{E_\zeta^{m\varphi}(1,\,2,\,\mathbf{H}_0)} = -\frac{gE_\theta^{ie\zeta}(2,\,1,\,-\mathbf{H}_0)}{\mathcal{H}_\varphi^{ie\zeta}(2,\,1,\,-\mathbf{H}_0)}$$
$$\frac{\mathcal{H}_\zeta^{e\theta}(1,\,2,\,\mathbf{H}_0)}{\mathcal{H}_\zeta^{m\varphi}(1,\,2,\,\mathbf{H}_0)} = -\frac{gE_\theta^{im\zeta}(2,\,1,\,-\mathbf{H}_0)}{\mathcal{H}_\varphi^{im\zeta}(2,\,1,\,-\mathbf{H}_0)} \qquad (16)$$

for the horizontal dipoles (magnetic oriented along the unit vector $\mathbf{e}_\theta$ and electric oriented along the unit vector $\mathbf{e}_\varphi$):

$$\frac{E_\zeta^{m\theta}(1,\,2,\,\mathbf{H}_0)}{E_\zeta^{e\varphi}(1,\,2,\,\mathbf{H}_0)} = -\frac{\mathcal{H}_\theta^{ie\zeta}(2,\,1,\,-\mathbf{H}_0)}{gE_\varphi^{ie\zeta}(2,\,1,\,-\mathbf{H}_0)}$$
$$\frac{\mathcal{H}_\zeta^{m\theta}(1,\,2,\,\mathbf{H}_0)}{\mathcal{H}_\zeta^{e\varphi}(1,\,2,\,\mathbf{H}_0)} = -\frac{\mathcal{H}_\theta^{im\zeta}(2,\,1,\,-\mathbf{H}_0)}{gE_\varphi^{im\zeta}(2,\,1,\,-\mathbf{H}_0)} \tag{17}$$

Here $E_\zeta^{q\xi}(1,\,2,\,\mathbf{H}_0)$ and $\mathcal{H}_\zeta^{q\xi}(1,\,2,\,\mathbf{H}_0)$ are the ζ components of the electric and magnetic fields, respectively, excited in the ionosphere at point 2 under the given geomagnetic field $\mathbf{H}_0$ by electric ($q = e$) or magnetic ($q = m$) dipoles oriented along the unit vector $\mathbf{e}_\xi$ and located in the waveguide cavity at point 1; $E_\xi^{iq\zeta}(2,\,1,\,-\mathbf{H}_0)$ and $\mathcal{H}_\xi^{iq\zeta}(2,\,1,\,-\mathbf{H}_0)$ are the ξ components of the fields excited at point 1 under the geomagnetic field $-\mathbf{H_0}$ by the electric or magnetic dipoles directed along $\mathbf{e}_\zeta$ and located in the ionosphere at point 2 (ξ and ζ take the values r, θ, and φ). Using expressions (12), we rewrite the ratios of the ionospheric field components (14)–(17) in the following form:

$$\mathbf{B}^{e\theta} = \frac{ikbR_\nu^{(e)\prime}(kb)}{\nu R_\nu^{(e)}(kb)}\mathbf{B}^{er}$$
$$\mathbf{B}^{mr} = \frac{\nu R_\nu^{(m)}(kb)}{ikbR_\nu^{(m)\prime}(kb)}\mathbf{B}^{m\theta}$$
$$\mathbf{B}^{e\theta} = ig\frac{R_\nu^{(e)\prime}(kb)}{R_\nu^{(e)}(kb)}\mathbf{B}^{m\varphi}$$
$$\mathbf{B}^{m\theta} = -\frac{iR_\nu^{(m)\prime}(kb)}{gR_\nu^{(m)}(kb)}\mathbf{B}^{e\varphi} \tag{18}$$

where $\mathbf{B}^{q\xi}$ ($q = e, m$; $\xi = r, \theta, \varphi$) are six-element column matrices of electric and magnetic field components.

[12] If the emitter is located on the surface of the ground ($b = a$), it follows from expressions (9) and (18) that

$$\mathbf{B}^{e\theta} = g\delta\mathbf{B}^{m\varphi}$$
$$\mathbf{B}^{e\varphi} = -g\delta\mathbf{B}^{m\theta}$$
$$\mathbf{B}^{e\theta} = \frac{ka\delta}{\nu}\mathbf{B}^{er}$$
$$\mathbf{B}^{mr} = \frac{\nu\delta}{ka}\mathbf{B}^{m\theta} \tag{19}$$

It is worth noting that in the case of a waveguide irregular on θ, the value δ in expressions (19) should be considered as the relative surface impedance of the ground in the regular part of the waveguide where the transmitter is located.

[13] Now we discuss the relationship between the components of the fields excited in the ionosphere by an emitter of any type located in the waveguide cavity. Methods and algorithms of field calculation in the waveguide cavity and in the ionosphere were developed in detail [*Rybachek*, 1995; *Rybachek et al.*, 1997a, 1997b], so we do not discuss these methods. As it has been already mentioned, determination of the fields in the ionosphere is reduced to an integration of the system of differential equations for the tangential components of the fields coinciding by the form with the equations describing propagation of plane waves in plane-stratified media. The difference is that the ionospheric properties and the complex angle of wave incidence on the ionosphere $\alpha(\nu, r)$ depend on the radial coordinate. The $\alpha(\nu, r)$ angle is determined by the relation $\sin\alpha(\nu, r) = \nu/(kr)$. The radial components of the fields are found from the Maxwell's equations for the ionospheric plasma. The asymptotic separation of variables in these equations leads to the field dependence on the coordinate θ of the type (8) [see, e.g., *Makarov et al.*, 1994], so the radial field components of each of normal waves are described by the following expressions:

$$E_r = -\frac{1}{\varepsilon_{rr}}\left[\varepsilon_{r\theta}E_\theta + \varepsilon_{r\varphi}E_\varphi + \frac{\nu}{kr}\mathcal{H}_\varphi\right] \tag{20}$$

$$\mathcal{H}_r = \nu E_\varphi/(kr) \tag{21}$$

Here $\varepsilon_{\xi\zeta}$ (ξ, $\zeta = r$, θ, φ) are the elements of the relative dielectric permittivity tensor of the ionosphere and r is the radial coordinate of the observation point.

[14] At low altitudes where the ionospheric properties are close to the properties of the free space, it follows from (20) that the components of the fields E_r and $\mathcal{H}_\varphi$ are related by a simple formula $krE_r \simeq -\nu\mathcal{H}_\varphi$. In the general case, the simple relation determined by formula (21) exists only between the components $\mathcal{H}_r$ and E_φ. Between the other components, there exists a complicated relation due, in particular, to a sphericity of the waveguide, inhomogeneity of the ionosphere, and its anisotropy. However, one can expect that at large distances from any emitter located near the ground surface, the fields in the ionosphere would be close to the fields of plane waves propagating in a magnetoactive plane-stratified medium. In this case the relation between the field components are considerably simplified.

[15] Actually, in the region of the ionosphere where the quasi-longitudinal approximation

$$|Y_\mathrm{T}^2| \ll 2|Y_\mathrm{L}(U - X)| \tag{22}$$

is valid the relations between the field components of a plane wave have the following form [*Budden*, 1961]:

$$E_\varphi/E_\theta = -\mathcal{H}_\theta/\mathcal{H}_\varphi = \pm i \tag{23}$$

$$\frac{E_r}{E_\theta} = \pm\frac{Y_\mathrm{T}X}{[(U \pm Y_\mathrm{L})(U - X)]} \tag{24}$$

$$\mathcal{H}_\varphi/E_\theta = n \qquad n^2 = 1 - X/(U \pm Y_\mathrm{L}) \tag{25}$$

The designations accepted in the magneto-ionic theory are used in formulae (22)–(25):

$$X = \frac{e^2N}{m_e\varepsilon_0\omega^2}$$

$$\mathbf{Y} = \frac{e\mu_0 \mathbf{H}_0}{m_e \omega} \tag{26}$$

$$U = 1 + i\frac{\nu_e}{\omega}$$

In (25) and (26): e, m_e are the charge and the mass of an electron, respectively, N is the electron concentration, ν_e is the effective collision frequency of electron with neutral particles and ions, n is the refractive index of the ionosphere, and Y_{L} and Y_{T} are the longitudinal and transverse components of the $\mathbf{Y}$ vector. In the case of a regular waveguide N and ν_e depend only on the radial coordinate. For an irregular waveguide these parameters depend in addition on the angular coordinate θ. Expressions (23)–(25) are written in the spherical coordinates with the aim to use them for the problems being studied.

[16] Relations (24) and (25) complicated enough because of the complex parameter U, are simplified if besides (22) the following inequalities are fulfilled:

$$|U| \ll |Y_{\mathrm{L}}| \ll X \tag{27}$$

In this case we have

$$E_\theta / E_r = -Y_{\mathrm{L}} / Y_{\mathrm{T}} \tag{28}$$

$$\mathcal{H}_\varphi / E_\theta = n \qquad n^2 = \mp X / Y_{\mathrm{L}} \tag{29}$$

It is worth noting that as it follows from formulae (26) and (29) for n^2, the values of the refractive indices may be either real or imaginary. Because of this, out of two propagating upward characteristic waves only one wave with the real n is of a significantly propagating character.

[17] It follows from expression (28) with an allowance for (26) that

$$|E_\theta / E_r| = |H_{0\mathrm{L}} / H_{0\mathrm{T}}| \tag{30}$$

where $H_{0\mathrm{L}}$ and $H_{0\mathrm{T}}$ are the longitudinal (radial) and transverse components of the geomagnetic field, respectively.

[18] Taking into account the relation which follows from (21) and (23)

$$|E_\theta| = |E_\varphi| = kr|\mathcal{H}_r/\nu|$$

and expression (29), we find

$$|\mathcal{H}_\theta / \mathcal{H}_r| = kr|n/\nu| \tag{31}$$

Here $|n|$ is computed from (26) and (29) as

$$|n| = c\sqrt{\frac{|e|N}{\omega |H_{0\mathrm{L}}|}} \tag{32}$$

where c is the free space velocity of electromagnetic waves.

4. Relations Between the Field Components Excited in the Waveguide by Ionospheric Dipoles

[19] The expressions relating the components of the fields excited in the terrestrial waveguide in point 2 with coordinates (r, θ) by various emitters located in the ionosphere in point 1 with coordinates (b, 0) can be obtained from the reciprocity theorems (1):
for the electric dipoles oriented along the unit vector $\mathbf{e}_\xi$, $\xi = \theta$, φ, and radial:

$$\frac{E_\zeta^{ie\xi}(1,\,2,\,\mathbf{H}_0)}{E_\zeta^{ier}(1,\,2,\,\mathbf{H}_0)} = \frac{E_\xi^{e\zeta}(2,\,1,\,-\mathbf{H}_0)}{E_r^{e\zeta}(2,\,1,\,-\mathbf{H}_0)}$$
$$\frac{\mathcal{H}_\zeta^{ie\xi}(1,\,2,\,\mathbf{H}_0)}{\mathcal{H}_\zeta^{ier}(1,\,2,\,\mathbf{H}_0)} = \frac{E_\xi^{m\zeta}(2,\,1,\,-\mathbf{H}_0)}{E_r^{m\zeta}(2,\,1,\,-\mathbf{H}_0)} \tag{33}$$

for magnetic dipoles oriented along the unit vector $\mathbf{e}_\xi$, $\xi = \theta$, φ, and radial:

$$\frac{E_\zeta^{im\xi}(1,\,2,\,\mathbf{H}_0)}{E_\zeta^{imr}(1,\,2,\,\mathbf{H}_0)} = \frac{\mathcal{H}_\xi^{e\zeta}(2,\,1,\,-\mathbf{H}_0)}{\mathcal{H}_r^{e\zeta}(2,\,1,\,-\mathbf{H}_0)}$$
$$\frac{\mathcal{H}_\zeta^{im\xi}(1,\,2,\,\mathbf{H}_0)}{\mathcal{H}_\zeta^{imr}(1,\,2,\,\mathbf{H}_0)} = \frac{\mathcal{H}_\xi^{m\zeta}(2,\,1,\,-\mathbf{H}_0)}{\mathcal{H}_r^{m\zeta}(2,\,1,\,-\mathbf{H}_0)} \tag{34}$$

Here $E_\zeta^{iq\xi}(1,\,2,\,\mathbf{H}_0)$ and $\mathcal{H}_\zeta^{iq\xi}(1,\,2,\,\mathbf{H}_0)$ are the ζ components of the intensities of the electric and magnetic fields, respectively, excited in the waveguide at the point 2 under the given geomagnetic field $\mathbf{H}_0$ by electric ($q = e$) or magnetic ($q = m$) dipoles oriented along the unit vector $\mathbf{e}_\xi$ and located in the ionosphere at point 1; $E_\xi^{q\zeta}(2,\,1,\,-\mathbf{H}_0)$, $\mathcal{H}_\xi^{q\zeta}(2,\,1,\,-\mathbf{H}_0)$ are the ξ components of the fields excited in the ionosphere at the point 1 under the geomagnetic field $-\mathbf{H}_0$ by electric or magnetic dipoles directed along $\mathbf{e}_\zeta$ and located in the waveguide at the point 2. In this case ζ takes the values r, θ, and φ.

[20] It follows from expressions (33) and (34) and relations (23), (30), and (31) that the formulae relating the components of the fields excited in the waveguide cavity by emitters of different types located in the ionosphere in the region, where quasi-longitudinal approximation (22) and inequalities (27) are applicable, can be written in matrix form as

$$\mathbf{A}^{ie\theta} = \mathbf{A}^{ie\varphi} \qquad \mathbf{A}^{im\theta} = \mathbf{A}^{im\varphi} \tag{35}$$

$$\mathbf{A}^{ie\theta} = \Re_e \mathbf{A}^{ier} \tag{36}$$

$$\mathbf{A}^{im\theta} = \Re_m \mathbf{A}^{imr} \tag{37}$$

Here $\mathbf{A}^{iq\xi}$ ($q = e, m; \xi = r, \theta, \varphi$) are six-element column matrices of the modules of the electric and magnetic field components and $\Re_e$ and $\Re_m$ according to formulae (30) and (31) are

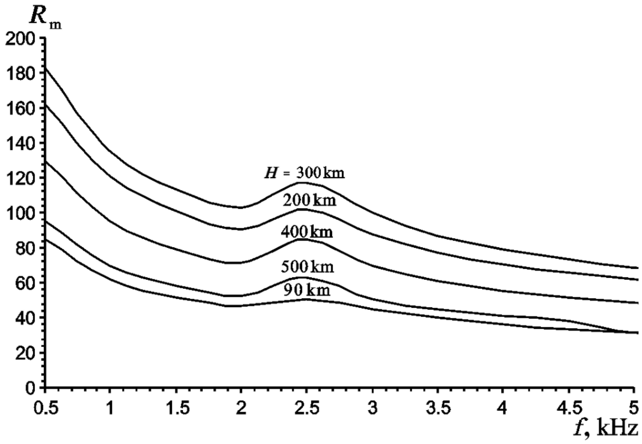

Figure 1. Frequency dependence of the efficiency parameter for different heights H of the source in the daytime ionosphere.

$$\Re_e = |H_{0\mathrm{L}}/H_{0\mathrm{T}}| \tag{38}$$

$$\Re_m = c\,\frac{kb}{|\nu|}\sqrt{\frac{|e|N}{\omega|H_{0\mathrm{L}}|}} \tag{39}$$

Here $b = a + H$, where H is the height of the emitter above the surface of the ground. It is seen that (39) may be written as

$$\Re_m \simeq 5.37\,\frac{kb}{|\nu|}\sqrt{\frac{N}{f|H_{0\mathrm{L}}|}}$$

where N and $H_{0\mathrm{L}}$ should be given in el cm^{-3} and oersteds, respectively.

[21] In the case of an irregular waveguide, the parameters in formulae (38) and (39) describing the ionospheric properties correspond to the point at which the emitter is located.

5. Efficiency of the Waveguide Excitation: Numerical Results and Interpretations

[22] The efficiency parameters of the waveguide excitation will be defined as

$$R_e = |E_r^{ie\theta}/E_r^{ier}| \tag{40}$$

$$R_m = |E_r^{im\theta}/E_r^{imr}| \tag{41}$$

where $E_r^{ie\theta}$ and E_r^{ier} are the radial components of the electric field excited on the surface of the ground by the ionospheric electric dipoles oriented along the unit vector $\mathbf{e}_\theta$ and radial, respectively. Similar values for magnetic dipoles are designated as $E_r^{im\theta}$ and E_r^{imr}. The field components in expressions (40) and (41) were calculated by the method described by *Rybachek* [1995] and *Rybachek et al.* [1997a, 1997b], and in the scope of the problem in question may be computed with any given accuracy. The values of R_e and R_m which in this sense may be called "accurate," as well as the approximate expressions for the efficiency parameters $\Re_e$ (38) and $\Re_m$ (39), following from (36) and (37), characterize the efficiency of excitation of the waveguide by horizontal and radial dipoles located in the ionosphere.

[23] The region where approximate expressions (38) and (39) are valid, is roughly described by inequalities (22) and (27). One can obtain more exact estimation introducing the relative errors

$$\varepsilon_e = |R_e - \Re_e|/R_e \tag{42}$$

$$\varepsilon_m = |R_m - \Re_m|/R_m \tag{43}$$

[24] In the case if the waveguide is a regular one on the coordinate θ the daytime and nighttime models of the electron concentration $N(H)$ [*Prikner*, 1980] and effective collision frequency of electron with neutral particles and ions $\nu_e(H)$ [*Fatkullin et al.*, 1981] were used for the calculations. The electrical properties of the ground correspond to the seawater (the specific conductivity is 4 mho m^{-1}). The calculations were performed for the frequencies of 0.5–5 kHz, the distance from the projection of the emitter onto the ground surface to the ground-based receiver of 500 km, the emitter location heights from 50 to 500 km, and latitudes $I = 10 - 80°$.

[25] The frequency dependencies of the efficiency parameter R_m (41) for various heights H of emitter location for the daytime and nighttime propagation conditions are shown in Figures 1 and 2, respectively ($I = 36°$). It follows from Figures 1 and 2 that at the heights of 100–500 km in the daytime and 200–500 km at night, the horizontal magnetic dipole is more effective than the vertical one ($R_m > 1$). Calculations show that for the heights of 50–60 km in the daytime and 50–80 km at night, R_m may be less than unity. In the daytime in the frequency range 2–3 kHz and at night at frequencies below 2 kHz, a nonmonotonic dependency $R_m(f)$ is observed.

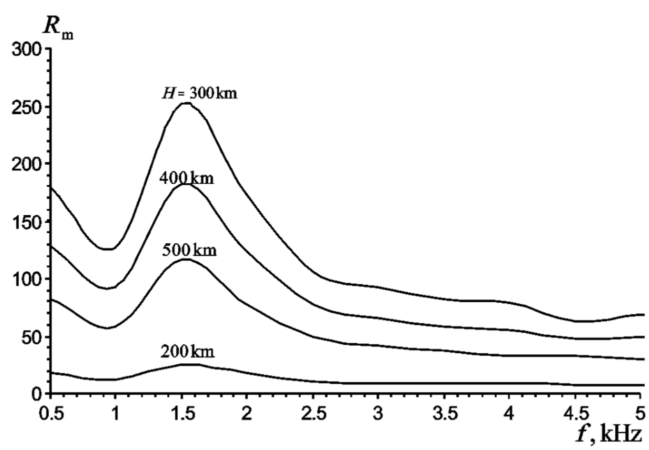

Figure 2. Frequency dependence of the efficiency parameter for different heights H of the source in the nighttime ionosphere.

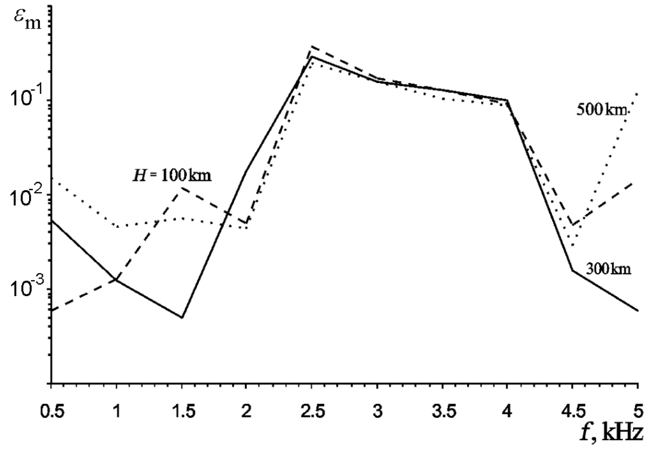

Figure 3. Frequency dependence of the relative error for daytime ionosphere ($H = 100$, 300, and 500 km).

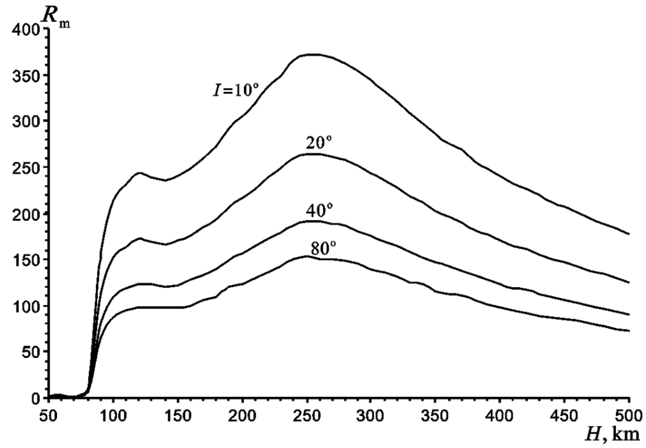

Figure 5. Dependence of the efficiency parameter R on the height H of the source for different latitudes in the daytime ionosphere ($f = 0.5$ kHz).

This is due to the presence of the so-called waveguide minima in the frequency dependencies of the field component modules. The position of the minima depends on the state of the ionosphere (day, night) and is different for different components of the fields. The efficiency parameter reaches the highest value $R_m \simeq 180$ in the daytime at a frequency of 0.5 kHz and $R_m \simeq 250$ at night at a frequency of $f \simeq 1.5$ kHz (at the source height of 300 km).

[26] Now we consider possibilities of using the approximate formula (39). For this purpose we calculated the frequency dependencies of the relative errors ε_m (43) for the daytime (Figure 3) and night (Figure 4). In the daytime at heights of 100–500 km, formula (39) is the most accurate at frequencies of 0.5–2 kHz (here $\varepsilon_m < 2\%$). At night, the use of approximation (39) is reasonable at the heights of 200–500 km and the same as for the daytime frequencies. That gives $\varepsilon_m < 10\%$.

[27] The dependencies of the efficiency parameter R_m on the height of the emitter locations ($H = 50 - 500$ km) for various latitudes I and a frequency of 0.5 kHz are shown in Figures 5 and 6 for the daytime and nighttime conditions, respectively. The ratio R_m reaches a maximum at all latitudes at $H \simeq 250$ and $H \simeq 300$ km in the daytime and at night, respectively. The ratio R_m takes the peak values of 350–370 at a latitude $I = 10°$. The corresponding daytime and nighttime dependencies of the relative error ε_m (43) on H for $I = 60°$ are shown in Figure 7. In the daytime at $H \geq 100$ km and at night at $H \geq 250$ km, the values of ε_m do not exceed 1%. The calculations show that for all considered latitudes in the daytime under $H \geq 100$ km, ε_m does not exceed 2%. At night under $H \geq 250$ km, the values of ε_m do not exceed 2%, whereas under $H \geq 200$ km, $\varepsilon_m \leq 12\%$. This fact makes it possible to use for interpretation of the results presented in Figures 5 and 6 approximate formula (39). According to this formula, the form of the $R_m(H)$ dependencies is determined by the features of the $N(H)$ profiles having a maximum at $H \simeq 250$ km in the

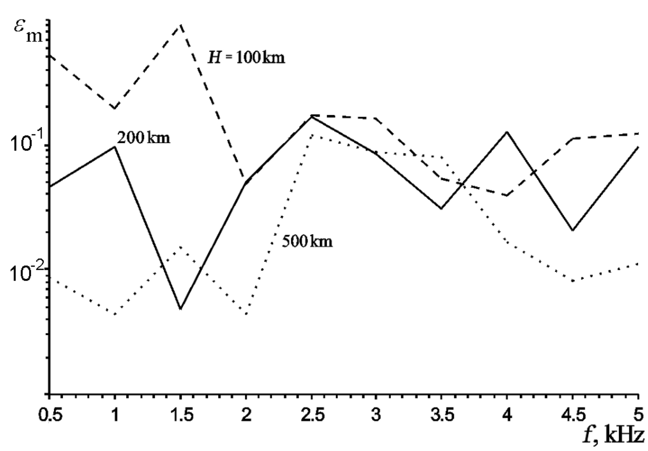

Figure 4. Frequency dependence of the relative error for nighttime ionosphere ($H = 200$, 300, and 500 km).

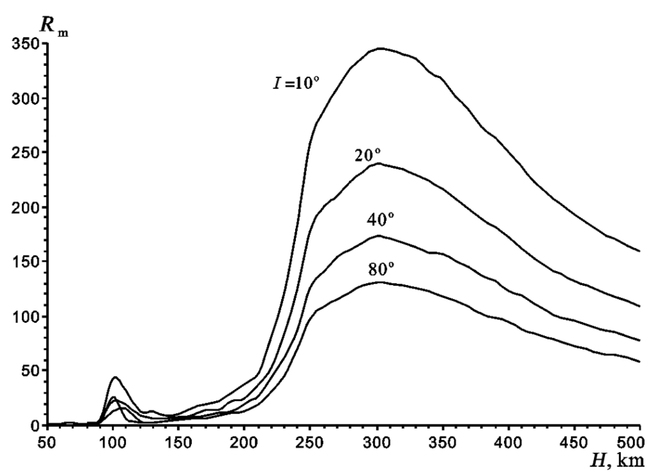

Figure 6. Dependence of the efficiency parameter R on the height H of the source for different latitudes in the nighttime ionosphere ($f = 0.5$ kHz).

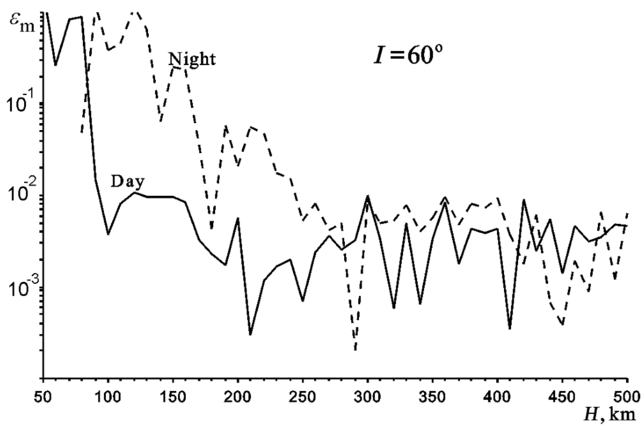

Figure 7. Dependence of the relative error on the height H for daytime and nighttime ionosphere ($f = 0.5$ kHz).

daytime and two maxima at $H \simeq 100$ and 300 km at night.

[28] The calculated by exact formulae dependences of the efficiency parameter R_e (40) on latitude I for the daytime and nighttime ionosphere, at a frequency of 0.5 kHz and altitudes of 100–500 km are described by the same curve in Figure 8. It follows from Figure 8 that at latitudes $I > 25°$ the horizontal electric dipole is more effective than the radial dipole, and the ratio $R_e \simeq 11.5$ at a latitude of 80°. The latitudinal dependencies of the relative errors ε_e (42) for altitudes of 100 and 500 km are shown in Figure 9 (solid lines correspond to the daytime values). In the daytime for all considered latitudes and emitter heights, it is valid: $\varepsilon_e < 0.2\%$. At night we have the same result for $H = 500$ km, whereas for $H = 100$ km at all latitudes $\varepsilon_e < 1.2\%$. Such small errors make it possible to use for interpretation of the results presented in Figure 8 approximate formula (38). According to this formula, the values of the efficiency parameters are determined only by the geomagnetic field, so we have one curve in Figure 8. This result

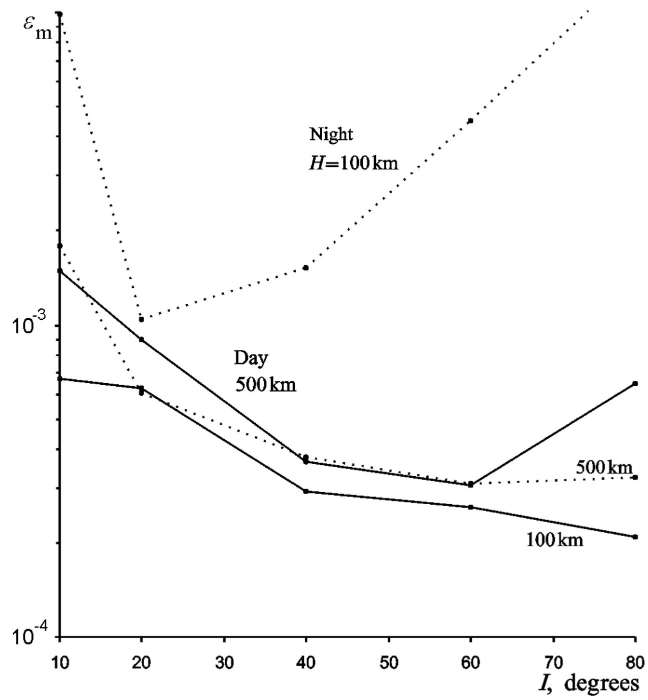

Figure 9. Dependence of the relative error on latitude for the heights $H = 100$ and 500 km for daytime and nighttime ionosphere ($f = 0.5$ kHz).

qualitatively is true at an increase of the frequency in the considered range. The calculations show that in this case for the daytime and nighttime midlatitude ionosphere and dipole location heights of 200–500 km, $\varepsilon_e < 5\%$.

[29] Similar results of calculations for an irregular waveguide are presented in Figure 10 in the form of the diurnal variations of the R_m parameter for a frequency of 0.5 kHz and the path with the emitter and receiver coordinates 30°N,

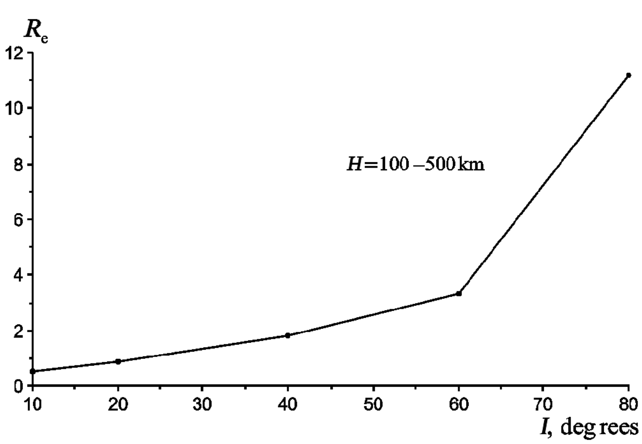

Figure 8. Dependence of the efficiency parameter R on latitude for the heights of the source $H = 100 - 500$ km for the daytime and nighttime ionosphere ($f = 0.5$ kHz).

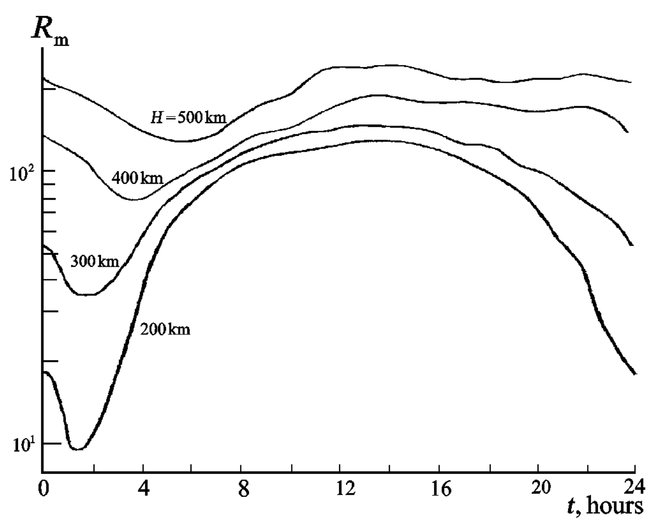

Figure 10. Daily variations of the efficiency parameter R for different heights H of the source ($f = 0.5$ kHz).

30°E and 60°N, 30°E, respectively. The data are obtained with the help of the algorithm [*Rybachek et al.*, 1997a, 1997b] taking into account the geomagnetic and geographic coordinates, solar zenith angle, azimuth, and local time. The path was approximated by seven regular parts. The $N(H)$ and $\nu_e(H)$ profiles from *Rawer et al.* [1978] and *COSPAR* [1990] and *Fatkullin et al.* [1981], respectively, were used in the calculations. It follows from Figure 10 that at the location of the transmitter at a height of 200 km, the daytime values of R_m ($t = 12$ h) exceed considerably the nighttime values ($t = 1$ h) (by about an order of magnitude). The excess is much less at altitudes of 300–500 km, this fact agreeing to the presented in Figures 1 and 2 results of calculations of the R_m parameter for the nighttime and daytime regular waveguide.

6. Conclusions

[30] Using the generalized reciprocity theorem, the expressions relating the components of the fields generated in the anisotropic ionosphere by various emitters located in the Earth–ionosphere waveguide and also the relations between field components excited in the waveguide by emitters located in the ionosphere are derived.

[31] In the frequency range 0.5–5 kHz in the scope of a regular terrestrial waveguide, parameters of the efficiency of the waveguide excitation by horizontal and radial electric and magnetic dipoles located in the ionosphere are studied. The analysis was performed using the numerical methods that allow us to derive the field components with the given accuracy and the approximate analytical formulae. The latter are valid in the case of the emitters location in the region of applicability of the quasi-longitudinal approximation.

[32] The analysis of the frequency dependencies of the R_m parameter characterizing the efficiency of waveguide excitation by magnetic dipoles showed that for the midlatitude ionosphere ($I = 36°$) in the considered ranges of frequencies and heights ($H = 100 - 500$ km and 200–500 km in the daytime and at night, respectively), a horizontal magnetic dipole is more efficient than a vertical one. The R_m parameter at a frequency of 0.5 kHz for $H = 300$ km is $R_m \simeq 180$ both in the daytime and at night. The use of the approximate formula for R_m is reasonable at frequencies of 0.5–2 kHz at altitudes of 100–500 km and 200–500 km in the daytime ($\varepsilon_m < 2\%$) and at night ($\varepsilon_m < 10\%$), respectively.

[33] The form of the R_m dependencies on the emitter height is determined by the features of the electron concentration profiles. In particular, the positions of the maxima in the dependencies $R_m(H)$ and $N(H)$ coincide at all latitudes. At a frequency of 0.5 kHz the efficiency parameter R_m reaches its maximum at all latitudes at $H \simeq 250$ and $H \simeq 300$ km in the daytime and at night, respectively. The value of R_m in the maximum can reach 350–370 (at a latitude of 10°). The accuracy of the approximate formula $\Re_m$ is high enough: for all considered latitudes at $H > 250$ km, ε_m is not more than 2% both in the daytime and nighttime conditions.

[34] The error of the approximate formula for the efficiency parameter R_e characterizing excitation of the waveguide by electric dipoles does not exceed 5% in the considered frequency range for the daytime and nighttime midlatitude ionosphere and dipole location heights of 200–500 km. In this case the efficiency of waveguide excitation by electric dipoles is determined mainly by the geomagnetic field.

[35] In the scope of an irregular waveguide, the diurnal dependencies of $R_m(t)$ are calculated using the algorithm taking into account geomagnetic and geographic coordinates, solar zenith angle, azimuth, and local time. For the midlatitudinal path at the location of the transmitters at altitude of 200 km and at a frequency of 0.5 kHz, the daytime values ($t = 12$ h) of R_m exceed considerably the nighttime values ($t = 1$ h) (by about an order of magnitude). This excess is much less at altitudes of 400–500 km.

[36] Concluding, we note that in the ELF range, the approximate formulae for the efficiency parameters make it possible, without calculating the fields in the ionosphere, with sufficiently high degree of accuracy to estimate the efficiency of the Earth–ionosphere waveguide excitation by emitters located at altitudes of 200–500 km both in the daytime and nighttime ionosphere.

References

Budden, K. G. (1961), *Radio Waves in the Ionosphere,* 542 pp., Cambridge Univ. Press, New York.

COSPAR, (1990), *International Reference Atmosphere,* 7, 519 pp., Elsevier, New York.

Einaudi, F., and J. R. Wait (1971a), Analysis of the excitation of the Earth–ionosphere waveguide by a satellite-borne antenna, *Can. J. Phys., 49,* 447.

Einaudi, F., and J. R. Wait (1971b), Analysis of the excitation of the Earth–ionosphere waveguide by a satellite-borne antenna, part II, *Can. J. Phys., 49,* 1452.

Erdelyi, A. (1953), *Higher Transcendental Functions, vol. 1,* 302 pp., McGraw-Hill, New York.

Fatkullin, M. N., T. I. Zelenova, A. F. Kozlov, V. K. Legen'ka, and T. N. Soboleva (1981), *Empirical Models of the Midlatitude Ionosphere* (in Russian), 256 pp., Nauka, Moscow.

Galejs, J. (1971a), Excitation of the terrestrial waveguide by sources in the lower ionosphere, *Radio Sci., 6,* 41.

Galejs, J. (1971b), VLF propagation across discontinuous day-time to night-time transitions in anisotropic terrestrial waveguide, *IEEE Trans. Antennas Propagat., 19*(6), 756, doi:10.1109/TAP.1971.1140034.

Ginzburg, V. L. (1970), *The Propagation of Electromagnetic Waves in Plasma,* 684 pp., Elsevier, New York.

Katsenelenbaum, B. Z. (1961), *The Theory of the Irregular Waveguides with Smoothly Variable Parameters* (in Russian), 216 pp., Nauka, Moscow.

Krasnushkin, P. E. (1962), On the propagation of long and very long radio waves around the Earth, *Nuovo Cimento, Suppl., 26,* 50.

Makarov, G. I., V. V. Novikov, and S. T. Rybachek (1994), *Radiowave Propagation in the Earth–Ionosphere Waveguide and in the Ionosphere* (in Russian), 152 pp., Nauka, Moscow.

Pappert, R. A. (1973), Excitation of the Earth–ionosphere waveguide by point dipoles at satellite heights, *Radio Sci., 8,* 535.

Prikner, K. (1980), Propagation of ULF electromagnetic waves through the ionosphere and geomagnetic pulsations, *Trav. Geophys., 543,* 143.

Rawer, K., D. Bilitza, and S. Ramakrishnan (1978), Goals and status of the international reference ionosphere, *Rev. Geophys., 16,* 177.

Rybachek, S. T. (1985), Electromagnetic fields of ionospheric point dipoles in the Earth–ionosphere waveguide, *Radiophysics* (in Russian), *28*(6), 703.

Rybachek, S. T. (1995), Radio wave propagation from antennae at satellite altitudes into the Earth–ionosphere waveguide, *J. Atmos. Terr. Phys., 57*(3), 303, doi:10.1016/0021-9169(94)E0002-5.

Rybachek, S. T., V. I. Ivanov, and V. L. Senina (1997a), Ionospheric fields excited by sources located in the Earth–ionosphere irregular waveguide, *J. Atmos. Sol. Terr. Phys., 59*(1), 139, doi:10.1016/1364-6826(95)00195-6.

Rybachek, S. T., V. I. Ivanov, and V. L. Senina (1997b), Excitation of the Earth–ionosphere irregular waveguide by sources located in an anisotropic ionosphere, *J. Atmos. Sol. Terr. Phys., 59*(5), 561, doi:10.1016/S1364-6826(96)00039-9.

M. Ju. Ponomariev and S. T. Rybachek, Institute of Radio Physics, St. Petersburg State University, 1/1 Ul'yanovskaya Str., Petrodvoretz 198504, St. Petersburg, Russia. (RYBACHEK@SR11107.spb.edu)