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Discussion paper: The eigen oscillations of the solar active regions

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[1] The standing MHD-waves in the potential magnetic arcades describing the bipolar active regions are investigated. The eikonal method allows us the analytical study of the short waves which are divided into the Alfvén and magnetosonic waves. The eigen modes of the magnetic arcades are formed as a result of their reflection from the photosphere. The Alfvén mode oscillations of the certain frequency take place on the magnetic surfaces. The fast mode oscillations also take place on some surfaces but they are not magnetic. Each oscillation surface has a discrete set of the eigen frequencies. *INDEX TERMS:* 7524 Solar Physics, Astrophysics, and Astronomy: Magnetic fields; 7836 Space Plasma Physics: MHD waves and instabilities; 7509 Solar Physics, Astrophysics, and Astronomy: Corona; *KEYWORDS:* Solar active regions; Eigen oscillations; Magnetic arcades.

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1. Eikonal Method for the Ideal MHD

[2] In the geometric acoustics short-wave perturbations are described by functions of the form $f(\mathbf{r},t) = A(\mathbf{r},t)\exp(iS(\mathbf{r},t))$, where A is called the wave amplitude and S is called the eikonal. For the monochromatic waves the eikonal has been taken as $S(\mathbf{r},t) = \omega(\tau(\mathbf{r}) - t)$. The eikonal method may be used when the wavelengths are small as compared with equilibrium state scale l_0. Let $v_{\mathrm{A}0}$ and c_{s0} be the scales of Alfvén and sound velocities, then the frequency should satisfy the following conditions $1/\omega \ll l_0/v_{\mathrm{A}0}$ and $1/\omega \ll l_0/c_{s0}$, i.e. the frequency should be high. The function $\tau(\mathbf{r})$ is also called the eikonal. For the MHD-waves it is derived from the equations

$$1 - \frac{(\mathbf{B}_0\nabla\tau)^2}{4\pi\rho_0} = 0 \qquad (1)$$

$$1 = \frac{1}{2}\left(\frac{\gamma P_0}{\rho_0} + \frac{\mathbf{B}_0^2}{4\pi\rho_0}\right)(\nabla\tau)^2 \pm$$

$$\frac{1}{2}\sqrt{\left(\frac{\gamma P_0}{\rho_0} + \frac{\mathbf{B}_0^2}{4\pi\rho_0}\right)^2(\nabla\tau)^4 - \frac{\gamma P_0}{\rho_0}\frac{(\mathbf{B}_0\nabla\tau)^2}{4\pi\rho_0}(\nabla\tau)^2}$$

where $P_0 = P_0(\mathbf{r})$, $\rho_0 = \rho_0(\mathbf{r})$, $\mathbf{B}_0 = \mathbf{B}_0(\mathbf{r})$ are the equilibrium state. The first equation corresponds to the Alfvén wave, the second gives the fast and slow magnetosonic waves. In the small plasma beta approximation the eikonal equation for the fast waves is

$$1 - \frac{\mathbf{B}_0^2}{4\pi\rho_0}(\nabla\tau)^2 = 0 \qquad (2)$$

The zero order approximation for the Alfvén waves is

$$\mathbf{v}_0 = A(\nabla\tau \times \mathbf{B}_0)$$

$$\mathbf{b}_0 = -A(\mathbf{B}_0\nabla\tau)(\nabla\tau \times \mathbf{B}_0) \qquad p_0 = 0 \qquad (3)$$

where A is some coefficient that is found from the solvability condition of the equations for the first order approximation

$$((\nabla \times \mathbf{b}_0) \times \mathbf{B}_0 + (\nabla \times \mathbf{B}_0) \times \mathbf{b}_0 -$$

$$(\mathbf{B}_0\nabla\tau)(\nabla \times (\mathbf{v}_0 \times \mathbf{B}_0)))(\nabla\tau \times \mathbf{B}_0) = 0$$

The zero order approximation for the magnetosonic waves is

$$\mathbf{v}_0 = A\left(D_\pm\nabla\tau - \frac{(\nabla\tau)^2}{4\pi\rho_0}(\mathbf{B}_0\nabla\tau)\mathbf{B}_0\right)$$

$$\mathbf{b}_0 = AD_\pm((\nabla\tau)^2\mathbf{B}_0 - (\mathbf{B}_0\nabla\tau)\nabla\tau)$$

$$p_0 = A\gamma P_0(\nabla\tau)^2\left(D_\pm - \frac{(\mathbf{B}_0\nabla\tau)^2}{4\pi\rho_0}\right)$$

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$$D_{\pm} = \frac{1}{2}\left(\frac{\gamma P_0}{\rho_0} + \frac{\mathbf{B}_0^2}{4\pi\rho_0}\right)(\nabla\tau)^2 \pm$$

$$\frac{1}{2}\sqrt{\left(\frac{\gamma P_0}{\rho_0} + \frac{\mathbf{B}_0^2}{4\pi\rho_0}\right)^2(\nabla\tau)^4 - \frac{\gamma P_0}{\rho_0}\frac{(\mathbf{B}_0\nabla\tau)^2}{4\pi\rho_0}(\nabla\tau)^2}$$

The equation for the coefficient A in case of the fast waves is

$$\Bigg(-\frac{1}{\rho_0}\nabla p_0 + \frac{1}{4\pi\rho_0}(\nabla\times\mathbf{b}_0)\times\mathbf{B}_0 +$$

$$\frac{1}{4\pi\rho_0}(\nabla\times\mathbf{B}_0)\times\mathbf{b}_0\Bigg)\times$$

$$\left(D_{\pm}\nabla\tau - \frac{(\nabla\tau)^2}{4\pi\rho_0}(\mathbf{B}_0\nabla\tau)\mathbf{B}_0\right) +$$

$$(\nabla\times(\mathbf{v}_0\times\mathbf{B}_0))\frac{D_{\pm}}{4\pi\rho_0}((\nabla\tau)^2\mathbf{B}_0 -$$

$$(\mathbf{B}_0\nabla\tau)\nabla\tau) - (\gamma P_0\nabla\mathbf{v}_0 + \mathbf{v}_0\nabla P_0)\times$$

$$\frac{(\nabla\tau)^2}{\rho_0}\left(D_{\pm} - \frac{(\mathbf{B}_0\nabla\tau)^2}{4\pi\rho_0}\right) = 0$$

Here plus and minus are taken for the fast and slow waves respectively. This results allow us to draw the conclusion that the short-wave perturbations in nonuniform mediums have the main properties of the MHD-waves in uniform mediums. The Alfvén and the magnetosonic waves are not coupled. The Alfvén and the slow waves do not propagate across the equilibrium magnetic field.

2. Oscillations of Magnetic Arcades

[3] Let us consider the bipolar solar active region with the potential magnetic field

$$\mathbf{B}_0 = B_0 e^{-x/l_0}\left(-\mathbf{e}_x \sin\frac{y}{l_0} + \mathbf{e}_y\cos\frac{y}{l_0}\right) \tag{4}$$

$0 < x < \infty$, $-\infty < z < \infty$, $-\pi/2 < y < +\pi/2$. The plasma density we take in the form

$$\rho_0(\mathbf{r}) = \rho_0 e^{-\delta x/l_0} \quad \delta \geq 0 \tag{5}$$

In the coronal conditions the small plasma beta approximation may be used. The plane $x = 0$ is the photosphere surface, $x > 0$ is the height above the photosphere, the parameter l_0 gives the active region scale. The magnetic field lines have the equation $e^{-x/l_0}\cos(y/l_0) = \cos(y_0/l_0)$ with some constant y_0 and formate the arches which are contained in the planes z =const.

[4] In this field there are the Alfvén waves propagating from one footpoint to another along the rays which coincide with the magnetic arches. Being reflected from the photosphere they form the standing waves that give the eigen Alfvén modes of the active region. The standing fast waves are formed on the rays similar to magnetic arches, their ends lie on the photosphere. The boundary condition on the photosphere is taken as

$$\mathbf{v}|_{x=0} = 0 \tag{6}$$

The standing waves are described by the symmetric and antisymmetric functions

$$\mathbf{v}(\mathbf{r},t) = \begin{cases} \mathbf{v}_0 e^{-i\omega t}\cos(\omega\tau(\mathbf{r})) & \text{even mode} \\ \mathbf{v}_0 e^{-i\omega t}\sin(\omega\tau(\mathbf{r})) & \text{odd mode} \end{cases} \tag{7}$$

The spectrum is

$$\omega = \frac{k_\tau}{\tau(0, y_0, z_0)} \tag{8}$$

where $k_\tau = (n+1/2)\pi$ for the even mode and $k_\tau = n\pi$ for the odd mode, the integer number $n \gg 1$. The expression for the eikonal $\tau(\mathbf{r})$ is defined below. The oscillations are localized near the rays whose footpoints are $(0, -y_0, z_0)$ and $(0, y_0, z_0)$. The rays form the surfaces, each of them is characterized by the discrete set of eigen frequencies. The frequencies vary continuously from one surface to another. For the Alfvén wave we obtain the eikonal

$$\tau(\mathbf{r}) = \frac{l_0}{v_{A0}}\left(e^{-x/l_0}\cos\frac{y}{l_0}\right)^{\delta/2-1}\Phi(y/l_0)$$

$$\Phi(\xi) = \begin{cases} \int_0^\xi(\cos\xi)^{-\delta/2}d\xi & \delta \neq 2 \\ \frac{l_0}{2v_{A0}}\ln\left(\frac{1+\sin(y_0/l_0)}{1-\sin(y_0/l_0)}\right) & \delta = 2 \end{cases}$$

Here $v_{A0} = B_0/\sqrt{4\pi\rho_0}$ is the Alfvén velocity at the corona basis. For the velocity and magnetic field amplitudes we have

$$\mathbf{v}_0(\mathbf{r}) = A\mathbf{e}_z$$

$$\mathbf{b}_0(\mathbf{r}) = -A\Phi'(y/l_0)\frac{B_0}{v_{A0}}\left(e^{-x/l_0}\cos\frac{y}{l_0}\right)^{\delta/2}\mathbf{e}_z$$

$$A = \sqrt{\Phi'(y/l_0)}G(e^{-x/l_0}\cos\frac{y}{l_0})$$

where $G(\xi)$ is an arbitrary function. The plasma motions are polarized along the tunnel of the magnetic arcade. The plasma distribution reveals itself in the space distribution of the eigen oscillations, for $\delta < 4$ they are localized close to the apexes of the magnetic arches, and for $\delta > 4$ they are localized close to the footpoints.

[5] For the fast waves we obtain the eikonal

$$\tau(\mathbf{r}) = \begin{cases} \frac{l_0}{v_{A0}}(\frac{\delta}{2}-1)e^{(\delta/2-1)x/l_0}\sin\frac{(\frac{\delta}{2}-1)y}{l_0} & \delta \neq 2 \\ \frac{y}{v_{A0}} & \delta = 2 \end{cases}$$

The amplitudes of the fast waves are

$$\mathbf{v}_0(\mathbf{r}) = -A\frac{1}{v_{A0}}e^{-(\delta/2-1)x/l_0}\times$$

$$\left(\mathbf{e}_x\cos\frac{y}{l_0} + \mathbf{e}_y\sin\frac{y}{l_0}\right)$$

$$\mathbf{b}_0(\mathbf{r}) = A\frac{B_0}{v_{A0}^2}e^{-(\delta-1)x/l_0}\times$$

$$\left(\mathbf{e}_x\cos\frac{(\delta/2-1)y}{l_0} + \mathbf{e}_y\sin\frac{(\delta/2-1)y}{l_0}\right)$$

$$A = \left(\cos\frac{(\delta/2-1)y}{l_0}\right)^{\delta/(\delta-2)}\times$$

$$G\left(e^{-(\delta/2-1)x/l_0}\cos\frac{(\delta/2-1)y}{l_0}\right)$$

The plasma motions are polarized in the plane of the magnetic arches. The fast rays obey the equation

$$e^{-(\delta/2-1)x/l_0}\cos((\delta/2-1)y/l_0) = \cos((\delta/2-1)y_0/l_0)$$

If $\delta < 2$, the rays rise infinitely upward in the corona and the fast waves propagate freely. In this case the frequency is arbitrary, i.e. the fast modes spectrum is continuous. If $\delta = 2$, the rays are transformed into a horizontal straight line. We can formulate no obvious boundary conditions in this case. If $\delta > 2$, the rays have the arch form and the waves may be reflected from the photosphere. Moreover, if $2 < \delta < 4$ the fast rays arches are more gently slopping than the magnetic arches. In this case only a part of the fast waves are localized close to photosphere, one part of the spectrum is discrete and the other part may be continuous. If $\delta > 4$, the fast rays arches are steeper than the magnetic arches, here all fast waves are localized and the spectrum is discrete fully.

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