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Discussion paper: The kink oscillations of the thin nonuniform coronal loops

B. B. Mikhalyaev

Kalmyk State University, Elista, Russia

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[1] MHD-oscillations of an inhomogeneous coronal loop consisting of a dense cord and a surrounding shell are investigated. Magnetic field in the cord is longitudinal and in the shell is azimuthal only. Usually the nonuniform field leads to the existence of resonance. However here we assume the resonance points non exist in the tube, i.e. the resonances are cutted. Our approach pursue a target — an investigation of an influence of the wave radiation on the tube oscillations. The resonant absorption of tube oscillation energy is eliminated. The same tube effectively radiate a magnetosonic waves into the environment and the Q-factor of the tube oscillations is small. The presented model can explain the fast damping of the coronal loop oscillations observed by the TRACE EUV channel. *INDEX TERMS:* 7509 Solar Physics, Astrophysics, and Astronomy: Corona; 7524 Solar Physics, Astrophysics, and Astronomy: Magnetic fields; 7836 Space Plasma Physics: MHD waves and instabilities; *KEYWORDS:* Coronal loops; Kink oscillations; Magnetosonic waves.

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[2] It is well known that the coronal magnetic flux tubes are twisted. In addition to the longitudinal magnetic field they also contain an azimutal component. The magnetic tubes undergoe expansion in the rarefied solar atmosphere. If the tube expands, the azimutal field $B_\varphi \sim r^{-1}$ on the periphery of the tube is formed. The longitudinal magnetic field persists only in the central part of the tube [*Parker*, 1979]. The mathematical difficulties that arise in describing of such coronal tube force us to use its crude model in which the magnetic field has only a longitudinal component in the central part and only an azimutal component on the periphery.

[3] We consider a cylindrical tube of radius a with the plasma density $\rho_{0m} = \rho_0/(\alpha r)^2$ in which a central part of radius b ($b < a$) (is called a cord) is detached by its plasma density ρ_{0i}, the other part of the tube is called a shell. The plasma density in the surrounding corona is $\rho_{0e} < \rho_{0i}$. The equilibrium magnetic field has the follow distribution

$$\mathbf{B}_0(r) = \begin{cases} B_{0i}\mathbf{e}_z & r < b \\ \dfrac{B_0}{\alpha r}\mathbf{e}_\varphi & b < r < a \\ B_{0e}\mathbf{e}_z & a < r \end{cases}$$

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The Alfvén speeds in the cord and in the corona are $V_{\rm Ai}, V_{\rm Ae}$. $V_{\rm Am}^2 = B_0^2/4\pi\rho_0$ is the Alfvén speed scale in the shell. The equilibrium conditions for the tube are

$$\rho_{0i}V_{\rm Ai}^2 = \frac{\rho_0 V_{\rm Am}^2}{\alpha^2 b^2}$$

$$\frac{\rho_0 V_{\rm Am}^2}{\alpha^2 a^2} = \rho_{0e}V_{\rm Ae}^2$$

[4] Let us seek the solutions of linear ideal MHD equations for the cool plasma in the form of cylindrical modes $f(r,t) = f(r)\exp(im\varphi + ik_z z - i\omega t)$, where k_z is the longitudinal wave number and ω is the frequency. They are expressed through the magnetic pressure perturbation $P(r) = p(r) + \mathbf{B}(r)\mathbf{B}_0(r)/4\pi$. The solution in the cord and in the corona can be expressed in terms of the Bessel equation solutions (for the kink-mode m=1):

$$v_{ri}(r) = \frac{-i\omega X_{0i}J_1'(k_i r)}{J_1'(k_i b)}$$

$$P_i(r) = \frac{X_{0i}\rho_{0i}(\omega^2 - V_{\rm Ai}^2 k_z^2)J_1(k_i r)}{k_i J_1'(k_i b)}$$

$$v_{re}(r) = \frac{-i\omega X_{0e}H_1^{(1)'}(k_e r)}{H_1^{(1)'}(k_e a)}$$

$$P_e(r) = \frac{X_{0e}\rho_{0e}(\omega^2 - V_{\rm Ae}^2 k_z^2) H_1^{(1)}(k_e r)}{k_e H_1^{(1)'}(k_e a)}$$

where

$$k_i^2 = \frac{\omega^2}{V_{\rm Ai}}^2 - k_z^2$$

$$k_e^2 = k_z^2 - \frac{\omega^2}{V_{\rm Ae}^2}$$

In the thin tube ($k_z a \ll 1$) approximation we take the principal terms of the Bessel and McDonald functions at $r = 0$. In the shell the radial equation for P have the form [*Appert et al.*, 1974]

$$\frac{d^2P}{d\zeta^2} + p(\zeta)\frac{dP}{d\zeta} + q(\zeta)P = 0$$

$$p(\zeta) = \frac{5}{\zeta} - \frac{2\lambda\zeta(2\lambda\mu\zeta^2 + \lambda - 6\mu)}{(\lambda^2\zeta^4\mu + \lambda^2\zeta^2 - 6\lambda\mu\zeta^2 - \lambda + \mu)} +$$

$$\frac{2\lambda\mu\zeta}{(\lambda\mu\zeta^2 + \lambda - \mu)}$$

$$q(\zeta) = -\nu + \frac{3}{\zeta^2} - \lambda\frac{-28\mu + 3\lambda + 4\lambda\mu\zeta^2 + \lambda^2\zeta^2}{\lambda^2\zeta^4\mu + \lambda^2\zeta^2 - 6\lambda\mu\zeta^2 - \lambda + \mu} +$$

$$\frac{-4\mu^2\nu + 4\nu\lambda\mu + 2\lambda^3}{(\lambda\mu\zeta^2 + \lambda - \mu)\lambda} + \frac{4\mu\nu}{(\lambda\zeta^2 - 1)\lambda}$$

Two linearly independent solutions of this equation have poles of the first and the third orders at $\zeta = 0$.

$$M(\zeta) \sim \frac{1}{\zeta} \qquad N(\zeta) \sim \frac{1}{\zeta^3}$$

The solution in the shell is

$$v_{rm}(r) = -i\omega\Bigg(\frac{C_1(r)}{C_3(r)}(FM(r) + GN(r)) +$$

$$\frac{D(r)}{C_3(r)}(FM'(r) + GN'(r))\Bigg)$$

$$P_m(r) = FM(r) + GN(r)$$

where F and G are arbitrary constants and the coefficients have the following approximate expressions

$$\frac{D}{C_3} \approx -\frac{\alpha^2 r^4}{\rho_0 V_{\rm Am}^2}$$

$$\frac{C_1}{C_3} \approx -\frac{2\alpha^2 r^3}{\rho_0 V_{\rm Am}^2}$$

Using the principals terms in the solutions and the boundary conditions

$$v_{ri}(b) = v_{rm}(b)$$

$$v_{rm}(a) = v_{re}(a)$$

$$P_i(b) = P_m(b) + \frac{B_{0\varphi}^2(b)}{4\pi i\omega b}v_{ri}(b)$$

$$P_m(a) + \frac{B_{0\varphi}^2(a)}{4\pi i\omega a}v_{ri}(a) = P_e(a)$$

we obtain the dispersion equation in the zeroth order approximation [*Mikhalyaev*, 2005]

$$\alpha^2 a^2 b^2 (a^2 - b^2)\rho_{0i}(\omega^2 - V_{\rm Ai}^2 k_z^2)\rho_{0e}(\omega^2 - V_{\rm Ae}^2 k_z^2) -$$

$$2\rho_0 V_{\rm Am}^2\{b^2\rho_{0i}(\omega^2 - V_{\rm Ai}^2 k_z^2) +$$

$$a^2\rho_{0e}(\omega^2 - V_{\rm Ae}^2 k_z^2)\} = 0$$

It should be borne in mind that the tube parameters in this equation are constrained by equilibrium conditions. This equation have two real solutions that describe undamped oscillations. One of them describes a fast magnetosonic wave whose phase speed exceeds the Alfvén speed in the corona. Therefore, it propagates radially into the surrounding corona, i.e., is radiated by the tube. The damping manifests itself as the effect of the next order with respect to the small $k_z a$. In the first approximation, the dispersion equation has a complex solution with a relatively small imaginary part. We write the complex frequency as $\omega = \omega_0(1+\epsilon)$, where ω_0 is the solution of the dispersion equation in the zeroth order approximation, and the dimensionless quantity ϵ gives the first correction. Its imaginary part determines the damping coefficient $-\omega_0 \,\mathrm{Im}\,\epsilon$, while the ratio $Q = -1/2\,\mathrm{Im}\,\epsilon$ is the Q-factor of the oscillations. For ϵ the following expression holds

$$8\omega_0^2\,\mathrm{Im}\,\epsilon\Bigg(\alpha^4 a^2 b^2 (a^2 - b^2)\rho_{0e}\rho_{0i}(2\omega_0^2 -$$

$$V_{\rm Ae}^2 k_z^2 - V_{\rm Ai}^2 k_z^2) - 2\rho_0 V_{\rm Am}^2(\alpha^2 a^2\rho_{0e} + \alpha^2 b^2\rho_{0i})\Bigg) +$$

$$\pi k_e^2 a^2\Bigg(\alpha^4 a^2 b^2 (a^2 - b^2)\rho_{0e}\rho_{0i}(\omega_0^2 - V_{\rm Ae}^2 k_z^2)\times$$

$$(\omega_0^2 - V_{\rm Ai}^2 k_z^2) -$$

$$2\rho_0 V_{\rm Am}^2(\alpha^2 a^2\rho_{0e}(\omega_0^2 - V_{\rm Ae}^2 k_z^2) -$$

$$\alpha^2 b^2 \rho_{0i}(\omega_0^2 - V_{\rm Ai}^2 k_z^2)) \Bigg) = 0$$

[5] We applied the results obtained to the oscillations of solar coronal loops. As the corona is characterized by Alfvén speeds much larger than the sound speed, we have chosen $V_{\rm Ae} = 700$ km s^{-1}. The density in the cord $\rho_{0i} = 5\rho_{0e}$, and the characteristic density $\rho_0 = 5\rho_{0e}$ was chosen for the shell. The scale parameter $\alpha = 0.25$ cm^{-1}. The Q-factor increases with decreasing wave number, i.e., with increasing tube length L. For example, at the tube radius a=12 Mm and the cord radius b=2 Mm, the Q-factor increases from 19.7 to 84.9 as the tube length changes from 11 Mm to 230 Mm. The oscillation period takes on values within the range 239 to 497 s. The Alfvén speed in the shell is the same, 939 km s^{-1}. The Q-factor and the period increase with cord radius. If b changes from 1 to 4 Mm (at $a = 12$ Mm and $L = 130$ Mm), then the period increases from 270 to 328, while the Q-factor increases from 18.1 to 190. Our calculations show that Q-factors close to their observed values can be obtained [*Nakariakov et al.*, 1999; *Ofman and Aschwanden*, 2002]. Thus, a double magnetic flux tube with a strongly twisted magnetic field in the shell can serve as an acceptable model for coronal loop, and the observed fast damping of transverse loop oscillations can be explained in terms of the effective radiation of fast magnetosonic waves into the surrounding corona by the loop.

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B. B. Mikhalyaev, Kalmyk State University, 11 Pushkin Str., Elista 358000, Russia.