INTERNATIONAL JOURNAL OF GEOMAGNETISM AND AERONOMY
VOL. 6, GI3009, doi:10.1029/2005GI000118, 2006

Electromagnetic field of the ground-based source surrounded by a plasma semispheroid

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Received 7 April 2005; revised 17 August 2006; accepted 24 August 2006; published 14 December 2006.

[1] Currently, in geophysics, there is growing interest about the deep layers of the Earth. Information about these layers may be obtained from analysis of the electromagnetic field at relatively low frequencies. Low-frequency antennas are usually very cumbersome. One aim is creation of an antenna that would be effective at low frequencies and compact enough. To do this, electromagnetic fields of ground-based sources in the presence of a plasma coating are studied. Such problems for the well-conducting Earth's surface are solved approximately using the equivalent theorem. This makes it possible to find the field at any distance from the source, including the Earth's surface. For a coating in the form of a two-layer plasma semispheroid, the solution is looked for in spheroidal functions attracting the theorem of addition. The influence of a plasma coating of high curvature is analyzed in the quasi-static approximation. *INDEX TERMS:* 2499 Ionosphere: General or miscellaneous; 2443 Ionosphere: Midlatitude ionosphere; 2427 Ionosphere: Ionosphere/atmosphere interactions; *KEYWORDS:* Earth's surface; Plasma; Resonance.

Citation: Bichoutskaia, T. I., and G. I. Makarov (2006), Electromagnetic field of the ground-based source surrounded by a plasma semispheroid, *Int. J. Geomagn. Aeron., 6,* GI3009, doi:10.1029/2005GI000118.

1. Introduction

[2] If the source has a plasma coating of the spherical or cylindrical shape [*Bichoutskaia and Makarov*, 1999; *Novikov and Soloviev*, 1996], it may lead to a considerable intensification of the radiation field (by 1–2 orders of magnitude) at resonance frequencies depending on the coating shape and plasma properties. The presence of a depleted ion layer around the source in the plasma [*Ratcliffe*, 1972; *Whale*, 1964] is able to enhance this intensification by several times [*Bichoutskaia and Makarov*, 2002, 2005; *Messiaen and Vandenplas*, 1967]. In this paper we study the influence on the radiation field of a ground-based source with the plasma coating of a more complicated spheroidal form, and also the value of the field for a changing form of the spheroid from a strongly prolate to a uniformly spherical and further to a compressed up to a strongly oblate spheroid form.

[3] The problem on the field of a ground-based source covered by a plasma semispheroid is considered in the presence of a flat boundary with the well-conducting Earth semispace. Such problem is reduced to the equivalent problem for the given source and its image in an ideally conducting plane (created using usual rules) located in the vacuum and surrounded by a plasma semispheroid and its reflection on the same plane. A vertical electric dipole and slot gap in a metal semispheroid are considered as the sources. In the equivalent problem the current at the antenna clamps or the voltage applied to the gap are doubled for these sources. We will consider these problems in a sequence and will call the resonance taking place in both cases as a resonance in current in the problem with a dipole and as a resonance in voltage in the problem with the slot antenna.

2. Formulation of the Problem for the Dipole Antenna

[4] We consider first an equivalent problem for a prolate plasma spheroid with the large and small semiaxes a and b and the eccentricity e located in the medium with the relative dielectric permeability ε_3. In the center of the spheroid an electrical dipole oriented along the rotation axis of the spheroid is located (Figure 1). We assume that the relative dielectric permeability of the cold isotropic plasma is

$$\varepsilon = 1 - \frac{\omega_p^2}{\omega^2 (1 + \dfrac{i\nu}{\omega})}$$

at the chosen time dependence of the form $\exp(-i\omega t)$, where

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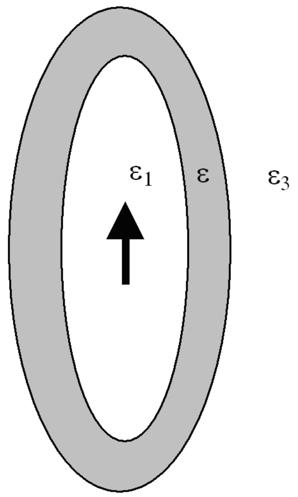

Figure 1. Geometry of a dipole antenna surrounded by a plasma spheroid.

ω_p is the circular plasma frequency of electrons and ν is the effective collision frequency.

[5] We assume that the depleted ion coating formed around the transmitter has a form of a spheroid with the semiaxes a_1 and b_1 and with the same eccentricity e. The spheroid is filled in by the medium with the relative dielectric permeability ε_1. The distance between the focal points of the inner and outer spheroids will be designated $2d_1$ and $2d$, respectively. Below we will take the electrical dimensions of the spheroids to be small, i.e., $ka \ll 1$ and $k\sqrt{|\varepsilon|}a \ll 1$, where k is the wave number in the vacuum.

[6] We will create the solution of the Maxwell equations in the outer medium ε_3 ($\operatorname{Im}\varepsilon_3 > 0$) satisfying the principle of radiation at the infinity using the system of spheroidal functions [*Morse and Feshbah,* 1953] determined in the prolate spheroidal coordinate system ξ, η, φ ($1 \le \xi < \infty$, $-1 \le \eta \le 1$) related to the outer spheroid

$$H_\varphi = \sum D_n S_{1n}(\bar{d}_{\varepsilon_3}, \eta) he^{(1)}_{1n}(\bar{d}_{\varepsilon_3}, \xi)$$

$$E_\eta = -\frac{i}{\varepsilon_3 \bar{d}} \sum D_n S_{1n}(\bar{d}_{\varepsilon_3}, \eta) h\dot{e}^{(1)}_{1n}(\bar{d}_{\varepsilon_3}, \xi) \tag{1}$$

Here and below the values of the magnetic and electric fields are multiplied by the impedance of the free space $Z_0 = \sqrt{\mu_0/\varepsilon_0}$ and by $\sqrt{\xi^2 - \eta^2}$, respectively. The following designations are used in (1): $S_{1n}(\bar{d}_{\varepsilon_3}, \eta)$ is the angular spheroidal function of the first kind and $he^{(1)}_{1n}(\bar{d}_{\varepsilon_3}, \xi)$ is the function related to the radial spheroidal functions of the first kind $je_{1n}(\bar{d}_{\varepsilon_3}, \xi)$ and second kind $ne_{1n}(\bar{d}_{\varepsilon_3}, \xi)$ by the formulae

$$he^{(1)}_{1n}(\bar{d}_{\varepsilon_3}, \xi) = je_{1n}(\bar{d}_{\varepsilon_3}, \xi) + i ne_{1n}(\bar{d}_{\varepsilon_3}, \xi)$$

$$h\dot{e}^{(1)}_{1n}(\bar{d}_{\varepsilon_3}, \xi) \equiv \frac{d}{d\xi}(\sqrt{\xi - 1} he^{(1)}_{1n}(\bar{d}_{\varepsilon_3}, \xi))$$

here $\bar{d}_{\varepsilon_3} = kd\sqrt{\varepsilon_3}$, and $\bar{d} = kd$. In the region of the depleted ion layer ε_1, we will create the solution using the system of functions determined in the coordinate system related to the inner spheroid:

$$H_\varphi = \sum S_{1n}(\bar{d}_{1\varepsilon_1}, \eta)[A_n he^{(1)}_{1n}(\bar{d}_{\varepsilon_3}, \xi) + R^1_n je_{1n}(\bar{d}_{1\varepsilon_1}, \xi)] \tag{2}$$

$$E_\eta = -\frac{i\bar{d}_1}{\bar{d}^2_{1\varepsilon_1}} \sum S_{1n}(\bar{d}_{1\varepsilon_1}, \eta)[A_n h\dot{e}_{1n}(\bar{d}_{1\varepsilon_1}, \xi) + R^1_n j\dot{e}_{1n}(\bar{d}_{1\varepsilon_1}, \xi)] \tag{3}$$

$$j\dot{e}_{1n}(\bar{d}_{1\varepsilon_1}, \xi) \equiv \frac{d}{d\xi}(\sqrt{\xi - 1} je_{1n}(\bar{d}_{1\varepsilon_1}, \xi))$$

here $\bar{d}_{1\varepsilon_1} = kd_1\sqrt{\varepsilon_1}$, and $\bar{d}_1 = kd_1$.

[7] In the region of the plasma layer with the relative dielectric permeability ε, we will create the solution using two systems of functions determined in the coordinate systems related to both inner and outer spheroids:

$$H_\varphi = H^{(1)}_\varphi + H^{(2)}_\varphi$$

$$E_\eta = E^{(1)}_\eta + E^{(2)}_\eta$$

$$H^{(1)}_\varphi = \sum D^1_n S_{1n}(\bar{d}_{1\varepsilon}, \eta) he^{(1)}_{1n}(\bar{d}_{1\varepsilon}, \xi)$$

$$E^{(1)}_\eta = -\frac{id_1}{\bar{d}^2_{1\varepsilon}} \sum D^1_n S_{1n}(\bar{d}_{1\varepsilon}, \eta) h\dot{e}^{(1)}_{1n}(\bar{d}_{1\varepsilon}, \xi) \tag{4}$$

$$H^{(2)}_\varphi = \sum R_n S_{1n}(\bar{d}_\varepsilon, \eta) je_{1n}(\bar{d}_\varepsilon, \xi)$$

$$E^{(2)}_\eta = -\frac{i\bar{d}}{\bar{d}^2_\varepsilon} \sum R_n S_{1n}(\bar{d}_\varepsilon, \eta) j\dot{e}_{1n}(\bar{d}_\varepsilon, \xi) \tag{5}$$

here $\bar{d}_{1\varepsilon} = kd_1\sqrt{\varepsilon}$, and $\bar{d}_\varepsilon = kd\sqrt{\varepsilon}$.

[8] The following designations are used in expressions (1)–(5): A_n are the multipliers of excitation of the field of the electric dipole in the unlimited space with the relative dielectric permeability ε_1, R^1_n and D^1_n are the reflection and transmission coefficients, respectively, for the boundary between the inner medium and the plasma coating, and R_n and D_n are the reflection and transmission coefficients, respectively, for the boundary between the plasma coating and the outer medium.

[9] The excitation coefficients A_n of the electromagnetic field of the source located in the center of the spheroid have the form

$$A_n = \frac{Gk\sqrt{\varepsilon_1} 2n(n+1)}{N_{1n}\bar{d}_{1\varepsilon_1}\sqrt{\xi_0^2 - 1}} je_{1n}(\bar{d}_{1\varepsilon_1}, \xi_0)$$

$$G = \frac{iZ_0}{4\pi} Ilk\sqrt{\varepsilon_1}$$

$$N_{1n} = \int_{-1}^{1} S_{1n}(\bar{d}_{1\varepsilon_1}, \eta)^2 d\eta$$

where ξ_0 is the radial coordinate of the source, I is the current at the transmitter entrance, and l is its effective length not exceeding the length of the rotation axis of the inner spheroid and satisfying the condition $kl\sqrt{\varepsilon_1} \ll 1$. According to *Morse and Feshbah* [1953], in the quasi-static approximation ($\bar{d}_{1\varepsilon_1} \ll 1$), one can choose among A_n the main coefficients corresponding to the index $n = 1$. Therefore one can limit expansions (2)–(3) by one spheroidal wave with $n = 1$.

[10] Writing the boundary conditions of continuity of the tangential components of the field on the surface of both spheroids and using for the spheroidal functions the theorem of addition [*Ivanov,* 1968] (which makes it possible to reexpand the solution of one system of functions in terms of another system of functions related to the outer spheroid and also to expand the angular spheroidal functions in the plasma medium on the angular functions of the inner or outer medium

$$S_{1l}(\bar{d}_{1\varepsilon}, \eta) = \sum_{n=1} B_{1n} S_{1n}(\bar{d}, \eta)$$

$$B_{1n} = \frac{1}{N_{1n}} \int_{-1}^{1} S_{1l}(\bar{d}_{1\varepsilon}, \eta) S_{1n}(\bar{d}, \eta) d\eta$$

we obtain an infinite system of algebraic equations. In the quasi-statistics condition, one can limit the angular function expansion with the accuracy of the terms of the order of $O[|\bar{d}_{1\varepsilon}|^2]$ and $O[|\bar{d}_\varepsilon|^2]$ by the main first term of the expansion with the coefficient $B_{11} = 1 + O[|\bar{d}_{1\varepsilon}|^2]$. Similar error is obtained using the first term in the expansion in the theorem of addition.

[11] Then confining ourselves by the main terms of the expansions we obtain the boundary conditions on the inner $\xi = \xi_1$ and outer $\xi = \xi_a$ surfaces of the plasma spheroidal layer in the form of a truncated system of four algebraic equations

$$A_n he^{(1)}_{1n}(\bar{d}_{1\varepsilon_1}, \xi_1) + R^1_n je_{1n}(\bar{d}_{1\varepsilon_1}, \xi_1) =$$

$$D^1_n he^{(1)}_{1n}(\bar{d}_{1\varepsilon}, \xi_1) + R_n je_{1n}(\bar{d}_{1\varepsilon}, \xi_1)$$

$$A_n h\dot{e}^{(1)}_{1n}(\bar{d}_{1\varepsilon_1}, \xi_1) + R^1_n j\dot{e}_{1n}(\bar{d}_{1\varepsilon_1}, \xi_1) =$$

$$\frac{\varepsilon_1}{\varepsilon}[D^1_n h\dot{e}^{(1)}_{1n}(\bar{d}_{1\varepsilon}, \xi_1) + R_n j\dot{e}_{1n}(\bar{d}_{1\varepsilon}, \xi_1)]$$

$$D^1_n he^{(1)}_{1n}(\bar{d}_\varepsilon, \xi_a) + R_n je_{1n}(\bar{d}_\varepsilon, \xi_a) = D_n he^{(1)}_{1n}(\bar{d}_{\varepsilon_3}, \xi_a)$$

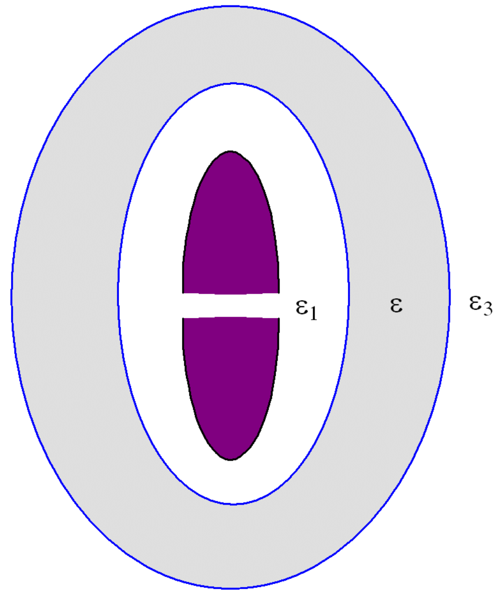

Figure 2. Geometry of a plasma-coated spheroidal antenna.

$$D^1_n h\dot{e}^{(1)}_{1n}(\bar{d}_\varepsilon, \xi_a) + R_n j\dot{e}_{1n}(\bar{d}_\varepsilon, \xi_a) = D_n \frac{\varepsilon}{\varepsilon_3} h\dot{e}^{(1)}_{1n}(\bar{d}_{\varepsilon_3}, \xi_a) \quad (6)$$

for $n = 1$. From equation system (6), we find the coefficient D_1 of the electromagnetic field propagation into the outer medium. Before finding expression for D_1, we will formulate the second equivalent problem in which a slot spheroidal antenna is used as an emitter.

3. Formulation of the Problem for a Slot Antenna

[12] Consider an ideally conducting slot antenna having the shape of a spheroid with the interfocal distance $2d_0$ and the large and small semiaxes a_0 and b_0, respectively, located in the medium with the relative dielectric permeability ε_3 and surrounded by a two-layer plasma coating (Figure 2). The outer surface of the first layer ε_1 (the depleted ion coating) is formed (in the same way as in the first problem) by the spheroid surface with the interfocal distance $2d_1$ and semiaxes a_1 and b_1. The second plasma layer ε is limited from the outside also by the spheroid surface with the interfocal distance $2d$ and semiaxes a and b. All three spheroids have the same origin of spheroidal coordinates and equal eccentricities e.

[13] The solution of the problem for an ideally conducting prolate spheroidal slot antenna put into a prolate two-layer plasma spheroid will be constructed in three systems of prolate spheroidal coordinates having the joint beginning. The slot antenna consists of two metal semispheroids ($\xi = \xi_0$) separated by a slot at $|\eta| < \Delta\eta$ ($\Delta\eta \ll 1$) to which the voltage V is applied. According to the commonly accepted statement [*Wait*, 1966], the tangential component of the electric

field of the slot antenna E_η, different from zero at the gap, is determined by the constant value independent of η

$$E_\eta = \begin{cases} 0 & \text{if } -1 < \eta < -\Delta\eta,\ -\Delta\eta < \eta < 1 \\ E_\eta^{a_0} & \text{if } -\Delta\eta < \eta < \Delta\eta \end{cases} \tag{7}$$

At a small width of the gap $2\Delta\eta$, this value is related to the voltage V by the formula

$$V = \int_{-\Delta\eta}^{+\Delta\eta} E_\eta(\xi_0, \eta) h_\eta d\eta \approx E_\eta^{a_0} \xi_0 2 d_0 \Delta\eta$$

The solution of the Maxwell equations in the outer medium ε_3 satisfying the principle of emission at the infinity will take the same form (1) as in the first problem.

[14] In the region of the depleted ion layer ε_1, the solution will be constructed (contrary to the first problem) using two systems of functions determined in the coordinate systems related to the inner and intermediate spheroids:

$$H_\varphi = H_\varphi^{(0)} + H_\varphi^{(1)}$$

$$E_\eta = E_\eta^{(0)} + E_\eta^{(1)}$$

$$H_\varphi^{(0)} = \sum A_n S_{1n}(\bar{d}_{0\varepsilon_1}, \eta) he_{1n}^{(1)}(\bar{d}_{0\varepsilon_1}, \xi)$$

$$E_\eta^{(0)} = -\frac{i\bar{d}_0}{\bar{d}_{0\varepsilon_1}^2} \sum A_n S_{1n}(\bar{d}_{0\varepsilon_1}, \eta) h\dot{e}_{1n}^{(1)}(\bar{d}_{0\varepsilon_1}, \xi) \tag{8}$$

$$H_\varphi^{(1)} = \sum R_n^1 S_{1n}(\bar{d}_{1\varepsilon_1}, \eta) je_{1n}(\bar{d}_{1\varepsilon_1}, \xi)$$

$$E_\eta^{(1)} = -\frac{i\bar{d}_1}{\bar{d}_{1\varepsilon_1}^2} \sum R_n^1 S_{1n}(\bar{d}_{1\varepsilon_1}, \eta) j\dot{e}_{1n}(\bar{d}_{1\varepsilon_1}, \xi) \tag{9}$$

here $\bar{d}_0 = kd_0$ and $\bar{d}_{0\varepsilon_1} = kd_0\sqrt{\varepsilon_1}$.

[15] In the region of the plasma layer with the relative dielectric permeability ε, the solution (in the same way as in the first problem) will take the form (4)–(5).

[16] At the presence of the boundary $\xi = \xi_1$ at the spheroid surrounding the slot antenna, to find A_n one should to equate the electrical components of the field (7), (8) and (9) on the surface of the inner spheroid $\xi = \xi_0$ preliminarily reexpanding system of functions (9) over the system of spheroidal functions (8) with the help of the theorem of addition. As a result, we obtain the infinite system of related algebraic equations. Confining ourselves by the main first term in the expansion with allowance for small parameters $|\bar{d}_{1\varepsilon_1}|$ and $|\bar{d}_{0\varepsilon_1}|$, we obtain the solution of the unrelated equation system in the form of the following relation for the coefficients of expansions (8) and (9)

$$A_n = G_n - R_n^1 f_{1n}$$

$$G_n = \frac{1}{N_{1n}} \frac{ik\varepsilon_1 V}{h\dot{e}_{1n}(\bar{d}_{0\varepsilon_1}, \xi_0)}$$

$$f_{1n} = \frac{j\dot{e}_{1n}(\bar{d}_{0\varepsilon_1}, \xi_0)}{h\dot{e}_{1n}(\bar{d}_{0\varepsilon_1}, \xi_0)} \tag{10}$$

where the normalizing multiplier for the angular functions N_{1n} for $n = 1$ has the representation

$$N_{11} \approx \frac{4}{3} + \frac{16}{75}\bar{d}_{0\varepsilon_1}^2 + \dots$$

at small $|\bar{d}_{0\varepsilon_1}| \ll 1$. The harmonics with $n > 1$ in (8) have the amplitudes A_n of the smaller order $O(|\bar{d}_{0\varepsilon_1}^{n+1}|)$ as compared to $n = 1$ and will not be taken into account below.

[17] Reexpanding in the same way the system of functions (5) over the system (4), presenting the angular functions of the plasma medium in the form of the expansion over the angular functions of the inner and outer medium, and confining ourselves by the main terms of the expansions, we obtain for the boundary conditions the same system of four algebraic equations (6) for $n = 1$, but with the A_n coefficients from (10).

4. Solution in Quasi-static Approximation for the Dipole and Slot Antennas

[18] For small electric dimension of the spheroid we take into account the following representation of radial spheroidal functions [*Morse and Feshbah*, 1953]

$$je_{11}(\bar{h}, \xi) = \frac{\bar{h}}{3}\sqrt{\xi^2 - 1}[1 + O(\bar{h}^2(\xi + \sqrt{\xi^2 - 1})^2)]$$

$$he_{11}(\bar{h}, \xi) = -\frac{3i}{2\bar{h}^2\sqrt{\xi^2 - 1}} \frac{\psi(\xi)}{\xi}(1 + i\Gamma) \times$$

$$[1 + O(\bar{h}^2(\xi + \sqrt{\xi^2 - 1})^2)] \tag{11}$$

$$\psi(\xi) = \xi^2 - \xi(\xi^2 - 1)\ \mathrm{arccth}\, \xi$$

$$\Gamma = \frac{2}{9}\bar{h}^3 \frac{\xi}{\psi(\xi)}(\xi^2 - 1)$$

Here the variables $\bar{h}$ and ξ take the values of the first and second arguments of spheroidal functions in (6). As a result of solution of system (6) we obtain for the transmission coefficients into the outer medium expressions containing explicit dependency on the problem parameters.

[19] For the prolate plasma spheroid surrounding a source with fixed current at the antenna clamps, we have the following expression for the propagation coefficient D_I of the leading harmonic

$$D_I = D_{0I} K_I$$

$$D_{0I} = iIl\frac{kZ_0}{4\pi}$$

$$K_I = \left\{\varepsilon(1+\Psi)^2[1+O(\bar{d}^2(\xi_a+\sqrt{\xi_a^2-1})^2)]\right\}\Big/$$

$$\left\{(1+\varepsilon\Psi)(\varepsilon+\Psi)-\alpha^3\Psi(1-\varepsilon)^2+\right.$$

$$\left. i\Gamma(1-\alpha^3)(1-\varepsilon)(\varepsilon+\Psi)\right\} \tag{12}$$

$$\Psi = \frac{1-\psi}{\psi} \qquad \Gamma = \frac{2}{9}\bar{a}^3\frac{1}{\psi}\left(\frac{b}{a}\right)^2 \qquad \alpha = \frac{a_1}{a}$$

For the prolate spheroidal antenna covered by a plasma sheath with the fixed voltage at the slot gap, the transmission coefficient D_V of the first harmonics has the form

$$D_V = D_{0V}K_V \qquad D_{0V} = \frac{kV}{N_{11}}\frac{\bar{b}_0^2}{3(1-\psi)} \qquad \bar{b}_0 = kb$$

$$K_V = \left\{\varepsilon[1+\Psi][F+\Psi+i\Gamma\alpha^3(F-1)]\times\right.$$

$$\left.[1+O(\bar{d}^2(\xi_a+\sqrt{\xi_a^2-1})^2)]\right\}\Big/$$

$$\left\{(\Psi+\varepsilon F)(1+\varepsilon\Psi)+\alpha^3\Psi(\varepsilon-1)(1-\varepsilon F)-\right.$$

$$\left. i\Gamma[(\varepsilon-1)(\Psi+\varepsilon F)+\alpha^3(1-\varepsilon F)(\varepsilon+\Psi)]\right\} \tag{13}$$

$$F = \Psi\frac{1-\alpha_0^3}{\Psi+\alpha_0^3} \qquad \alpha_0 = \frac{a_0}{a_1}$$

where the fact is taken into account that the depleted ion layer of the coating and the outer medium are vacuum: $\varepsilon_1 = \varepsilon_3 = 1$. The geometrical parameters α_0 and α characterize the relative thickness of the depleted ion and plasma coatings, respectively.

[20] We will call the functions K_I (12) and K_V (13), what at the absence of the plasma coating become equal to unity, functions of influence of the two-layer plasma coating on current and voltage.

[21] In the case of a oblate plasma spheroidal coating for the same sources considered above, the solution of the boundary problem constructed in oblate spheroidal variables using oblate angular and radial spheroidal functions leads for the $\tilde{D}_I$ and $\tilde{D}_V$ transmission coefficients to expressions similar to (12) and (13) where $\psi(\xi)$, Γ, and D_{0V} should be replaced by

$$\tilde{\psi}(\xi) = -\xi^2 + \xi(\xi^2+1)\ \mathrm{arccot}\,\xi$$

$$\tilde{\Gamma} = \frac{2}{9}\bar{a}^3\frac{1}{\tilde{\psi}}\left(\frac{b}{a}\right)$$

$$\tilde{D}_{0V} = \frac{kV}{N_{11}}\frac{\bar{a}_0^2}{3(1-\tilde{\psi})}$$

These substitutions should be performed always transferring from the prolate shape of the spheroid to the oblate one, so below we will present the expression only for the prolate shape of the spheroid.

[22] The value of D_{0V} characterizing the radiation field of a slot antenna without a coating located in the vacuum reaches the maximum value for the spherical shape of the slot antenna with the radius a_0 ($\psi = 2/3$), because for strongly prolate spheroidal antenna this value decreases as

$$\frac{\bar{a}_0^2}{3|\ln\frac{b_0}{a_0}|}$$

whereas for strongly oblate one (when $\bar{a}_0^2/3(1-\tilde{\psi}) \approx \bar{a}_0^2/3$) it appears by a factor of three less than for the spherical one.

[23] It is worth noting that for the dipole antenna located within a plasma spheroid, expression (12) does not allow for the limiting transition to the case of a one-layer plasma coating ($\alpha = 0$) [*Bichoutskaia and Makarov*, 2002, 2005]. At similar transition ($\alpha_0 = 1$) expression (13) becomes equal to the expression for the transmission coefficient for the one-layer plasma coating of the slot antenna.

[24] We substitute transmission coefficients (12) and (13) into the components of the electric field (1) in the outer medium. For the wave zone of the source $|\bar{d}_{\varepsilon_3}\xi| \gg 1$ we use asymptotics of radial spheroidal functions [*Morse and Feshbah*, 1953]

$$he_{11}^{(1)}(\bar{d}_{\varepsilon_3},\xi) = \frac{1}{\bar{d}_{\varepsilon_3},\xi}\exp(i\bar{d}_{\varepsilon_3},\xi) + O\left(\frac{1}{\xi^2}\right)$$

and assuming $\varepsilon_3 = 1$ we come to the electromagnetic field components in the vacuum in a spherical coordinate system.

$$E_r = \frac{2i}{kr}\frac{\exp(ikr)}{r}\cos\theta D$$

$$E_\theta = \frac{\exp(ikr)}{r}\sin\theta D$$

$$H_\varphi = \frac{\exp(ikr)}{r}\sin\theta D \tag{14}$$

where D has the value of (12) or (13) for the problem with an electric dipole or slot antenna.

[25] According to (14) the tangential component of the field E_r on the Earth's surface ($\cos\theta = 0$) becomes zero, as it should be for the infinite conducting Earth's surface. On the finitely conducting underlying surface at such distances the wave by its structure is close to a plain one, and the tangent component of the field may be obtained from the vertical component $E_r \approx [1/\sqrt{\varepsilon(x,y)}]E_\theta$ which is determined from formula (14) multiplied by the attenuation function containing properties of the propagation path.

5. Resonant Frequencies

[26] Now we study some features of the character of the dependence of the functions of influence K_I (12) and K_V (13) on the changing shape and relative thickness of two layers of the plasma coating. It is worth noting that because of the dispersion properties of the plasma one can find a frequency at which the values of $|K_I|$ and $|K_V|$ reach the maximum (resonance) value. At small losses to emission ($\Gamma \ll 1$) and small thermal losses in the plasma ($\mathrm{Im}\varepsilon \ll |\mathrm{Re}\varepsilon|$) when $\mathrm{Re}\varepsilon \approx 1 - \omega_p^2/\omega^2$ such resonant frequency in current is determined from the condition of becoming zero of the real part of the denominator (12)

$$(\Psi + \mathrm{Re}\,\varepsilon)(1 + \Psi \mathrm{Re}\,\varepsilon) -$$

$$\alpha^3 \Psi (\mathrm{Re}\,\varepsilon - 1)^2 = 0 \tag{15}$$

or the real part of the denominator (13) for the resonance in voltage

$$(\Psi + F\mathrm{Re}\,\varepsilon)(1 + \Psi \mathrm{Re}\,\varepsilon) +$$

$$\alpha^3 \Psi (\mathrm{Re}\,\varepsilon - 1)(1 - F\mathrm{Re}\,\varepsilon) = 0 \tag{16}$$

Equations (15) and (16) are biquadratic relative to the resonant frequency. The roots of the each equation form two branches at a change of the spheroid shape Ψ from strongly prolate to strongly oblate.

[27] The roots of equation (15) (at $\alpha > 0$) have the form

$$\frac{\omega^2}{\omega_p^2} = \frac{1}{2}(1 \pm \sqrt{1 - 4\psi^*})$$

$$\psi^* = \psi(1-\psi)(1-\alpha^3) \tag{17}$$

and form two branches depending on the spheroid shape b/a at its fixed filling in by the vacuum α.

[28] At continuous changes in the spheroid shape from strongly prolate one ($\Psi \approx 1,\ \Psi^* \approx 0$) to strongly oblate one ($\tilde{\Psi} \approx 1,\ \tilde{\Psi}^* \approx 0$), the resonant frequency (17) varies from the exit point (the resonance frequency at $b/a = 0$ for the prolate spheroid) to the coinciding to it entrance point (the resonance frequency at $b/a = 0$ for the strongly oblate spheroid) equal to $\omega^{(1)} \approx 0$ and $\omega^{(2)} \approx \omega_p$ for the low-frequency and high-frequency branches, respectively. It is worth noting that for a small filling in of the plasma spheroid by vacuum ($\alpha^3 \ll 1$), the approximate expressions for resonant frequencies (17) for the low-frequency ($\omega^{(1)}$) and high-frequency branches ($\omega^{(2)}$) for the prolate spheroid

$$\omega^{(1)} \approx \omega_p \sqrt{1-\psi}$$

and

$$\omega^{(2)} \approx \omega_p \sqrt{\psi}$$

and for the oblate spheroid

$$\omega^{(1)} \approx \omega_p \sqrt{\tilde{\psi}}$$

and

$$\omega^{(2)} \approx \omega_p \sqrt{1-\tilde{\psi}}$$

do not describe their values for the oblate shape of the spheroid in the vicinity of $\tilde{\Psi} = 1/2$ ($a \approx 2b$), where the resonant frequencies (17) have an extreme

$$\left(\frac{\omega}{\omega_p}\right)^{(1)}_{\mathrm{extr}} = \sqrt{\frac{1-\alpha\sqrt{\alpha}}{2}}$$

$$\left(\frac{\omega}{\omega_p}\right)^{(2)}_{\mathrm{extr}} = \sqrt{\frac{1+\alpha\sqrt{\alpha}}{2}} \tag{18}$$

These values (18) tend to the entrance (or exit) points of its branches with an increase in the filling in α of the spheroid by vacuum, that is, the curves of the dependence (17) on the spheroid shape b/a become less steep. In more detail for the fixed α, the position of resonance frequencies (17) as a function of b/a for the prolate spheroid and of the inverse parameter a/b for the oblate spheroid is shown in Figure 3a. Solid and dashed curves show the low-frequency and high-frequency branches, respectively, for two values of the relative thickness of the plasma coating: $\alpha = 0.2$ and 0.8. The calculation result illustrate the shift of the resonance frequency to the limiting values $\omega^{(1)} = 0$ and $\omega^{(2)} = \omega_p$ as the vacuum cavity becomes larger in volume ($\alpha = 0.8$). For the small vacuum filling in ($\alpha = 0.2$) resonance frequencies (17) have a well-pronounced extreme for the oblate shape of the spheroid.

[29] In the case of the slot antenna, the roots of equation (16) for the resonant frequencies have a form of complicated radicals, so their simple analytical dependence on the problem parameters may be obtained only in two particular cases: $F = 1$ and $F = 0$.

[30] For $F \approx 1$ we have a two-layer plasma coating of a weakly prolate spheroidal shape with a relatively thick first layer ($\alpha_0^3 \ll 1, \alpha_0^3 \ll \Psi$), and equation (16) coincide with resonant equation (15), the roots of which are presented by expression (17). Thus, in this case, the frequencies for the resonances in voltage and current coincide. These resonant frequencies are shown in Figure 3a.

[31] At the absence of the depleted ion layer ($F = 0, \alpha_0 = 1$), equation (16) is a resonance one for the spheroidal slot antenna with one-layer plasma coating ε. The roots of (16) have one frequency branch

$$\omega = \omega_p \sqrt{1-\psi^*}$$

to which (as we will see below) the resonant frequencies of the high-frequency branch (16) tend with a decrease of the thickness of the first layer ($\alpha_0 \to 1$). These frequencies are shown in Figure 3b for two values of the relative thickness of the plasma coating: $\alpha = 0.2$ and 0.8.

[32] For the α_0 values different from zero and unity, the regularities in the behavior of the resonant frequency (18) (related to ω_p) are shown in Figure 3c as a function of the spheroid shape by solid and dashed curves for the low-frequency and high-frequency branches, respectively, for

three values of the relative thickness of the first layer α_0. Curves 1 and 2 correspond to $\alpha_0 = 0.2$, $\alpha = 0.2$ and 0.8; curves 3 and 4 correspond to $\alpha_0 = 0.8$, $\alpha = 0.2$ and 0.8; and curves 5 and 6 correspond to $\alpha_0 = 0.99$, $\alpha = 0.2$ and 0.8.

[33] For the relatively thick first layer ($F \approx 1$, $\alpha_0^3 \ll 1$) the dependence of the resonant frequencies in voltage (curves in Figure 3c) coincide with the corresponding dependence for the resonance in current (curves that correspond to $\alpha = 0.2$ and 0.8 in Figure 3a).

[34] In the case when the thickness of the first layer becomes too small ($F \approx 0$, $\alpha_0 \approx 1$) the dependence of the resonance frequencies of the high-frequency branch (dashed curves 5 and 6 in Figure 3c) on the spheroid shape coincide with the corresponding dependence for the spheroidal slot antenna with a one-layer plasma coating (curves 0.2 and 0.8 in Figure 3b). The branch of lower resonance frequencies (solid curves 5 and 6 in Figure 3c) shifts to the region $\omega \approx 0$. Thus with a decrease of the thickness of the first layer of the coating, the branch of higher resonance frequencies transfers into the resonance frequencies for a slot antenna with an one-layer plasma coating, whereas the branch of lower resonance frequencies disappears. Therefore one may take that for the resonance in voltage the high-frequency and low-frequency branches characterize a resonance of the plasma with the outer and inner vacuum, respectively. We will draw the final conclusion after evaluation of the value of the function of influence for the both branches of resonant frequencies.

6. Resonant Function of Influence

[35] We estimate at resonance frequencies the value of the function of influence in current (12), which at the resonant condition (15) (taking into account small thermal losses in the plasma) in a main (zero) approximation over small parameter (electrical dimension of the spheroid) has the form

$$|K_I| = \frac{1}{\psi^* \left(1 - \dfrac{1}{\varepsilon_{\mathrm{Re}}}\right)} \times$$

$$\frac{1}{\left| \varepsilon_{\mathrm{Im}} \left(1 + \dfrac{1}{\varepsilon_{\mathrm{Re}}}\right) - \Gamma_s \left(\dfrac{\varepsilon_{\mathrm{Re}} \psi}{1 - \psi} + 1\right) \right|} \tag{19}$$

where $\varepsilon_{\mathrm{Re}} = 1 - \omega_p^2/\omega^2$, $\varepsilon_{\mathrm{Im}} = \nu_p(\omega_p^3/\omega^3)$, and $\nu_p = \nu/\omega_p$. The analytical dependence of (19) on the spheroid shape Ψ and its filling in by the vacuum α is rather complicated because of the presence of radicals in (17), except some limited region of the values of Ψ and α which we will consider below.

[36] The estimation of the resonant value of the function of influence at the extreme frequency (13) typical for an oblate spheroid gives the following dependence on the relative thickness of the plasma layer

$$|K_I^{(1,2)}| = \frac{\sqrt{1 - \alpha^3}}{2\sqrt{2}\alpha^{3/2}} \times$$

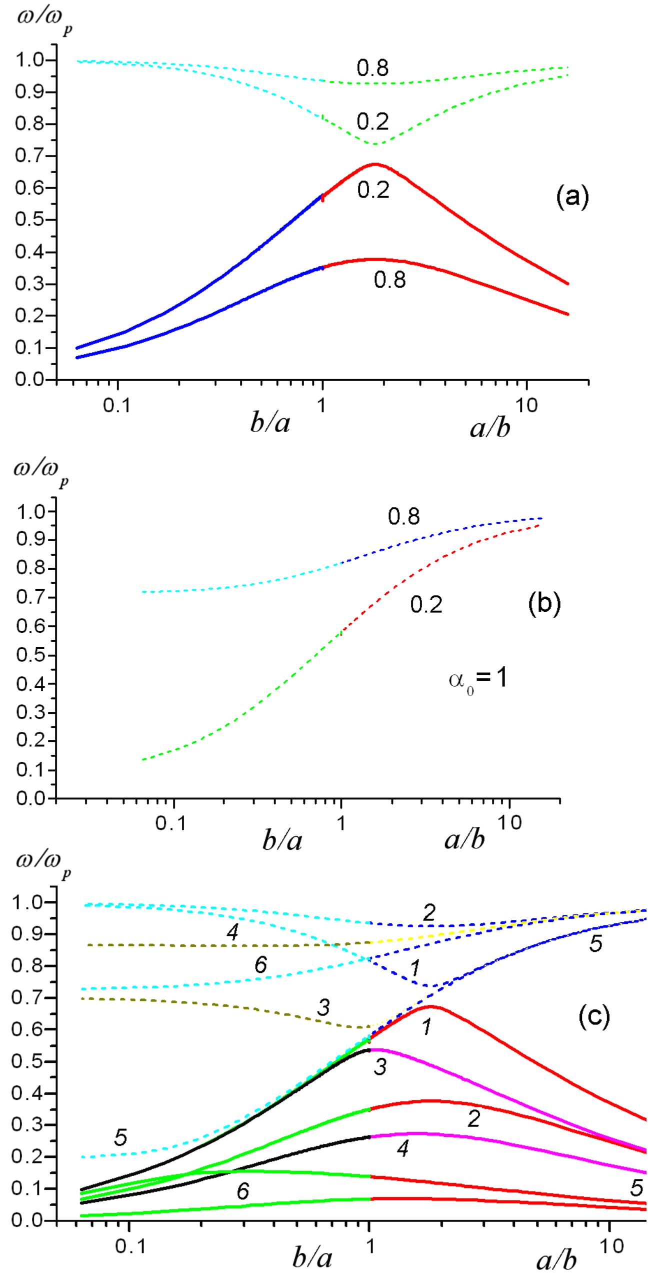

Figure 3. Branches of high (dashed curves) and low (solid curves) resonant frequencies as a function of b/a (prolate spheroid) and of a/b (oblate spheroid) for $\alpha = 0.2$ and 0.8 for (a) a dipole surrounded by two-layer plasma spheroid, (b) a plasma-coated spheroidal antenna with $\alpha_0 = 1$, and (c) a two-layer plasma-coated spheroidal antenna for $\alpha_0 = 0.2$ ($\alpha = 0.2$ (curve 1) and $\alpha = 0.8$ (curve 2)), $\alpha_0 = 0.8$ ($\alpha = 0.2$ (curve 3) and $\alpha = 0.8$ (curve 4)), and $\alpha_0 = 0.99$ ($\alpha = 0.2$ (curve 5) and $\alpha = 0.8$ (curve 6)).

$$\frac{\sqrt{1 \pm \alpha^{3/2}}}{\nu_p + \dfrac{1}{18}\bar{a}_p^3 \dfrac{b}{a}(1 - \alpha^3)(1 \mp \alpha^{3/2})} \tag{20}$$

With a decrease of the filling in of the oblate plasma spheroid by the vacuum cavity α, the value (20) increases considerably and exceeds the resonant coefficient of propagation for a sphere with a cavity [*Bichoutskaia and Makarov*, 2005]. Therefore the oblate shape of the plasma spheroid appears more preferable than the spherical one at the resonance in current.

[37] One can see in more detail the dependence of the resonant function of influence in current (19) on the spheroid shape b/a and its filling in α by vacuum in Figure 4a, where the function $|K_I|$ is shown as a function of the spheroid shape by solid and dashed curves for the low-frequency and high-frequency branches at two values of the second layer thickness: $\alpha = 0.2$ and 0.8. The function $|K_I|$ in Figure 4a is normalized to the modulus of the resonant function of influence in current $|K_{I0}|$ of the plasma sphere with radius α surrounding the electrical dipole at the resonant frequency $\omega_p/\sqrt{3}$

$$|K_{I0}| = \frac{1}{\nu_p\sqrt{3} + \dfrac{2\sqrt{3}}{27}\bar{a}_p^3} \qquad \bar{a}_p \equiv a\frac{\omega_p}{c} \tag{21}$$

The value $|K_{I0}|$ (21) exceeds the unity by 1 or 2 orders of magnitude depending on how small are the loss parameters ν_p and $\bar{a}_p^3$. We assumed at calculations that the terms in the denominator of (21) are equal, i.e., $\nu_p\sqrt{3} = (2\sqrt{3}/27)\bar{a}_p^3 = 0.01$.

[38] Because of the complexity of expression (13), one can obtain the explicit analytical dependence of the resonant function in voltage K_V on the problem parameters only at $F = 1$ and $F = 0$.

[39] In the case when a spheroidal coating is not strongly prolate and has a relatively thick first layer ($\alpha_0^3 \ll 1$, $\alpha_0^3 \ll \Psi$, $F \approx 1$), the resonant function $|K_V|$ (13) appears close to the resonant function of influence in current $|K_I|$.

[40] In the case $F = 0$ ($\alpha_0 = 1$, i.e., there is no first layer), the function K_V (13) is a resonance function of influence in voltage of a one-layer plasma coating of the spheroidal slot antenna [*Bichoutskaia and Makarov*, 2003]

$$|K_{V1}| = \frac{1}{\dfrac{\nu_p}{\psi^*}\sqrt{1-\psi^*} + \Gamma\dfrac{1-\psi^*}{1-\psi}}$$

$$\Gamma = \frac{2}{9}\frac{\bar{a}_p^3}{\psi}\Big(\frac{b}{a}\Big)^2(1-\psi^*)^{3/2} \tag{22}$$

The function decreases monotonously with a change of the spheroid shape Ψ from strongly prolate to strongly oblate one. This dependence of the function $|K_{V1}|$ normalized to (21) is shown in Figure 4f for two values of the thickness of the plasma layer $\alpha_0 = 0.2$ and 0.8.

[41] For the decreasing relative thickness of the first layer ($0.2 \le \alpha_0 < 1$) the resonant function $|K_V|$ (13) normalized to (21) versus the changing shape of the spheroid is shown in Figure 4b, Figure 4c, Figure 4d, and Figure 4e by solid and dashed curves for the low-frequency and high-frequency resonances, respectively, at two values of the relative thickness of the plasma layer $\alpha = 0.2$ and 0.8.

[42] For the relatively thick first layer with $\alpha_0 = 0.2$, the function $|K_V|$ (curves 0.2 and 0.8 in Figure 4b) does not differ from the resonant function of influence in current $|K_I|$ (curves 0.2 and 0.8 in Figure 4a), having for the oblate spheroid shape ($\alpha \approx 2b$) the maximum value exceeding by an order of magnitude the resonant function for sphere $|K_{I0}|$. With a decrease of the first layer thickness for $\alpha_0 = 0.8$ and $\alpha_0 = 0.9$ (curves 0.2 in Figures 4c and 4d) the extreme value of $|K_V|$ (almost not changing in magnitude) shifts into the region of more prolate spheroid shape. In the case of the spherical shape of the plasma coating, the value of the resonant function of influence $|K_V|$ may considerably exceed the value $|K_I|$, the latter statement following from the comparison at $a = b$ of the values of these functions shown by curves 0.2 in Figures 4c and 4a. At further depletion of the thickness of the first layer ($\alpha_0 = 0.9$ and $\alpha_0 = 0.999$), the high-frequency resonance $|K_V^{(2)}|$ (dashed curves in Figures 4d and 4e) is transformed into the resonance $|K_{V1}|$ of a one-layer plasma coating (curves 0.2 and 0.8 in Figure 4f). The latter resonance has a maximum value at the strongly prolate spheroid shape, for which the resonant value of the field is not so high because of low values of the excitation coefficient D_{0V}.

[43] One can show that at a small relative thickness of the first layer $\delta\Psi \ll 1$, ($\delta = 1 - \alpha_0^3$) for the branch of lower resonant frequencies in voltage (16)

$$\left(\frac{\omega^{(1)}}{\omega_p}\right)^2 \approx \hat{\delta}\frac{\psi^*}{1-\psi^*}$$

$$\hat{\delta} = \delta(1-\psi)$$

the modulus of the function of influence (13)

$$|K_V^{(1)}| \approx \frac{\sqrt{\hat{\delta}\psi^*}}{\nu_p(1-\psi^*)^{3/2}}$$

decreases down to zero with a decrease of the thickness of the first layer. We will take the low-frequency resonance vanishing at the first layer thickness $\hat{\delta}_0 \approx (\nu_p^2/\psi^*)(1-\psi^*)^3$, at which the value $|K_V^{(1)}|$ becomes equal to unity.

[44] The modulus of the function of influence (13) $|K_V^{(2)}|$ for the branch of higher resonance frequencies (16) is close to the modulus of the resonance function of influence $|K_{V1}|$ of a one-layer plasma coating (22). This is confirmed by the comparison of dashed curves in Figures 4e and 4f.

[45] Thus, with a decrease of the thickness of the first layer of the plasma coating, the low-frequency resonant function in voltage $|K_V^{(1)}| \approx 0$ disappears, whereas the high-frequency function $|K_V^{(2)}|$ tends to $|K_{V1}|$. Therefore one can interpret the low-frequency resonance in voltage as a resonance of the plasma with the inner vacuum unlike the resonance in current [*Bichoutskaia and Makarov*, 2002, 2005].]

7. Radiation Resistance and Input Impedance

[46] The expression for the power P^{rad} and resistance R^{rad} of radiation of the source with the given current at the antenna clamps

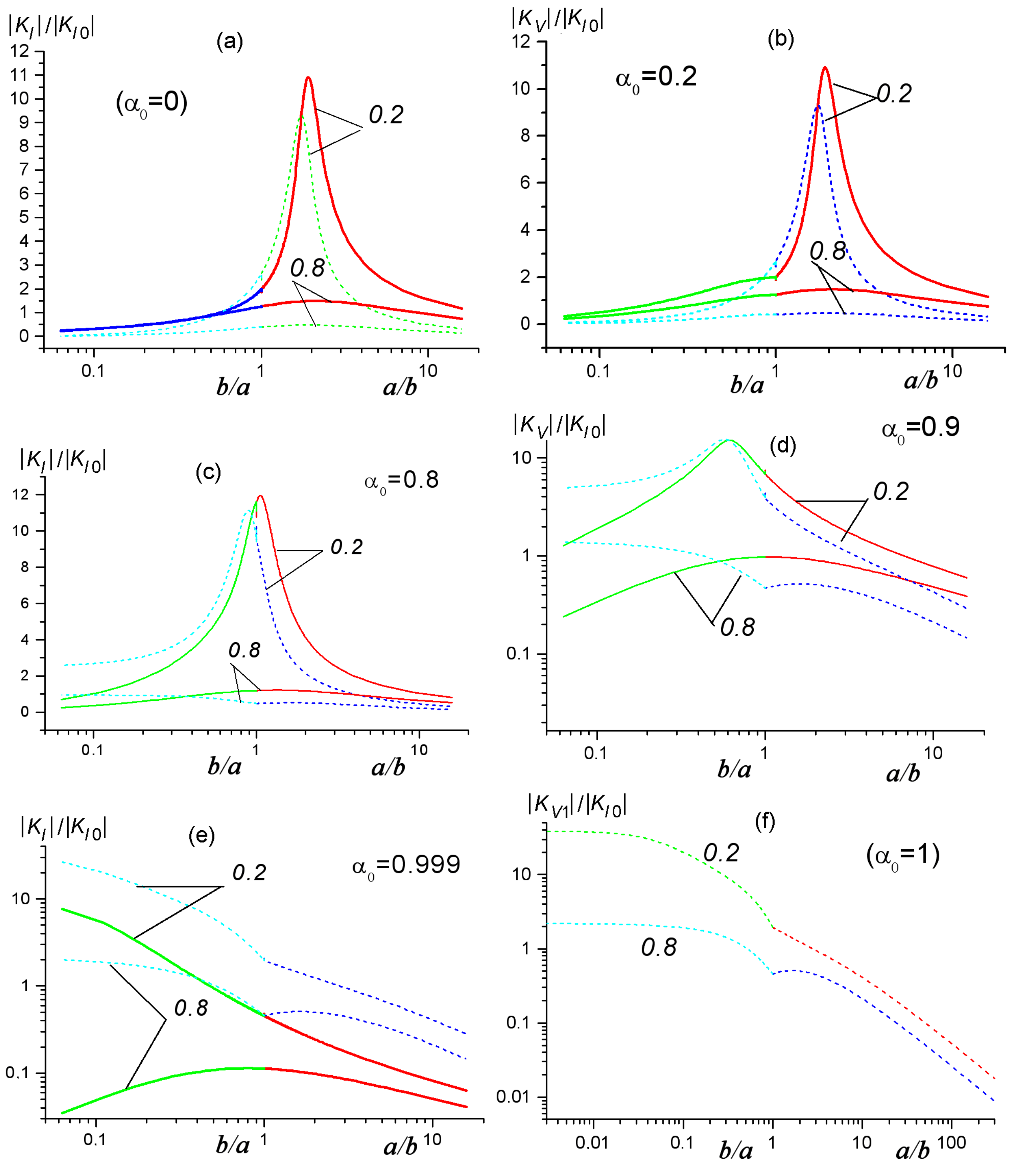

Figure 4. High- (solid curves) and low-frequency (dashed curves) resonant functions of influence versus b/a (prolate spheroid) and of a/b (oblate spheroid) for $\alpha = 0.2$ and 0.8 for two-layer plasma spheroid surrounding the dipole and two-layer plasma coating of spheroidal antenna (a) at $\alpha_0 = 0$, (b) $\alpha_0 = 0.2$, (c) $\alpha_0 = 0.8$, (d) $\alpha_0 = 0.9$, (e) $\alpha_0 = 0.999$, and (f) $\alpha_0 = 1$.

$$P^{\mathrm{rad}} = \pi r^2 \int_0^\pi E_\theta H_\varphi^* \sin\theta d\theta \Big|_{r\to\infty} = \frac{1}{2} I^2 R^{\mathrm{rad}}$$

leads according to (14) and (12) to the following expression for the resistance of this source with a plasma coating

$$R^{\mathrm{rad}} = R_0^{\mathrm{rad}} |K_I|^2$$

where $R_0^{\mathrm{rad}} = 20(kl)^2$ is the resistance of the radiation of the source without a coating. This expression shows that the resistance or power of the radiation at the resonance at the same conducted current increases according to (20) by 2–4 orders of magnitude if the function of influence increases by 1–2 orders of magnitude.

[47] The estimate of the input impedance Z of the emitter on the basis of the results of equations (6) solution relative to the reflection coefficient R_n^1 shows that the ratio $Z_{\mathrm{Im}}/Z_{\mathrm{Re}}$ not at the resonance is high $O(1/\Gamma\alpha^3)$. At the resonance frequency, this ratio having the value $O(\nu_p)$ decreases by almost 4 orders of magnitude if $\Gamma \approx \nu_p \approx 0.01$.

[48] The estimate of the power P^{rad} and conductivity Y^{rad} of the radiation of a spheroidal slot antenna surrounded by a two-layer plasma coating

$$P^{\mathrm{rad}} = \pi r^2 \int_0^\pi E_\theta H_\varphi^* \sin\theta d\theta \Big|_{r\to infty} = \frac{1}{2} V^2 Y^{\mathrm{rad}}$$

taking into account expressions (14) for the tangential components of the field in the wave zone of the source, shows that the radiation power

$$P^{\mathrm{rad}} = \frac{4\pi}{3Z_0 k^2} |\hat{D}_0|^2 |K_V|^2$$

contains the function of influence squared. At the resonant frequency at fixed voltage at the antenna gap, each component of the field according to (13) increases by 1–2 orders of magnitude (and in some cases by 3 orders). The power or conductivity of the radiation increases by 2–4 orders of magnitude (or more), being maximal for some shape of the spheroid and relative thickness of the plasma layers.

[49] The value of the complex entrance conductivity estimated on the basis of the H_φ component of the field at the antenna gap under very thin first plasma layer ($F \approx 0$ or $\alpha_0^3 \approx 1$) shows that for the low-frequency branch the ratio $|\mathrm{Im}Y^{(1)}|/\mathrm{Re}Y^{(1)} \approx O(\nu_p/\sqrt{\hat{\delta}})$ is small by the magnitude at the first layer thickness $\sqrt{\hat{\delta}} \gg \nu_p$ providing the presence of a considerable resonance. The ratio $\mathrm{Im}Y^{(2)}/\mathrm{Re}Y^{(2)} \approx O(\nu_p/\alpha^3)$ for the high-frequency branch is also small at not too thick second plasma layer.

8. Conclusions

[50] Thus the plasma coating of a dipole antenna at fixed current at its clamps (or of a slot antenna at fixed voltage at its gap) at some frequencies promote a considerable increase of the field in vacuum and an increase of $\mathrm{Re}Z_{\mathrm{in}}$ ($\mathrm{Re}Y_{\mathrm{in}}$ for a slot antenna) and leads to a relative decrease in $\mathrm{Im}Z_{\mathrm{in}}$ ($\mathrm{Im}Y_{\mathrm{in}}$), the latter fact making easier the coordination of the generator to the load. The resonant value of the field exceeds almost by 3 orders of magnitude the field of the antenna without a coating.

[51] The shape of the spheroidal slot antenna with a two-layer plasma coating providing the maximum resonant impact on the radiation field depends on the thickness of the first layer. With a decrease of the first layer thickness, this shape changes from the oblate one ($a/b \approx 2$) to the prolate one typical for a one-layer plasma coating. If there is no coating, the maximum radiation field would be at a slot antenna of a spherical shape.

[52] It is shown that use of a plasma coating of a dipole or slot ground-based antenna under some conditions is able to improve the coordination of the generator to the load and to lead to an increase of the low-frequency field at the Earth's surface. That would increase the reliability of the information on the deep layers of the Earth.

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