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# Energetic characteristics of radiation of oscillating dipoles travelling in some dispersive and moving media

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[1] Radiation of an oscillating electric dipole which is travelling with a constant velocity in homogeneous isotropic medium with given dispersion characteristic is studied. The cases of cold plasma and the medium with the dispersion of a resonant type are considered. It is shown that in the case of the resonantly dispersive medium, the radiation spectrum consists (depending on the problem parameters) either of two separated frequency ranges or of one frequency range. Regularities characterizing the dependence of the radiated power on the source motion velocity at various values of the resonant and plasma frequencies are derived. The case when the dipole travels in a moving medium the velocity of which is parallel or antiparallel to the source motion velocity is also considered. It is noted in particular that at some parameters of the problem, the energy loss of the source are negative. *INDEX TERMS:* 6934 Radio Science: Ionospheric propagation; 7839 Space Plasma Physics: Nonlinear phenomena; 2435 Ionosphere: Ionospheric disturbances; *KEYWORDS*: Cold plasma emission; Oscillating dipoles; VLF radioemission.

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## 1. Introduction

[2] The problem on radiation of sources moving in homogeneous stationary media has been actively studied during many decades. The main attention was drawn to the analysis of the radiation of charged particles and their beams and also sources having this or that multipole moment. The results obtained in this field are presented in many monographs and papers [see, e.g., *Afanasiev and Kartavenko*, 1998; *Afanasiev et al.*, 1999; *Bolotovskiy*, 1957; *Carusotto et al.*, 2001; *Frank*, 1981; *Ginzburg*, 1987, 2002; *Stevens et al.*, 2001; *Zrelov*, 1968]. It should be noted, however, that in the major part of the problems considered earlier the source was assumed to be static in its own reference system (i.e., it has no "proper" frequency). In such case (if one can neglect the irregularity in the source motion) the radiation at the given frequency exists only in the situation when the source motion velocity exceeds the phase velocity of the electromagnetic waves of this frequency (the Vavilov–Cerenkov radiation). In the case of a moving source oscillating in the "proper" reference system, the situation becomes principally different. Naturally, such source radiates at any velocity of the motion, but the radiation characteristics depend on it in a very significant way. Such problems are of an interest both for development of the theory and for experiments in various regions of physics. In particular, they present an interest for the analysis of radiation processes in the ionospheric plasma. In this case the antenna of a spacecraft, for example, may serve as an oscillator of the dipole type. A radiating moving atom presents another example of this kind.

[3] Various aspects of the theory of the radiation of oscillators moving in some simple media have been considered in publications [see, e.g., *Frank*, 1942, 1981; *Ginzburg and Frank*, 1947a, 1947b; *Tyukhtin,* 2004a]. This paper is dedicated to the analysis of the influence of medium characteristics on the radiation power of the oscillator and its spectral density. So we will restrict ourselves by the consideration of the case of an oscillating electric dipole uniformly moving in the direction of its dipole moment. Such formulation of the problem makes it possible to reveal principal physical regularities and, at the same time, to avoid too cumbersome mathematical expressions.

[4] Section 2 of this paper contains general expressions applicable to motionless medium with an arbitrary frequency dispersion. The main features of the cold plasma case are also noted. Section 3 is dedicated to the case of the medium with the dispersion of a resonant type. The situation when not only the oscillator is moving but the surrounding medium as well is considered in section 4.


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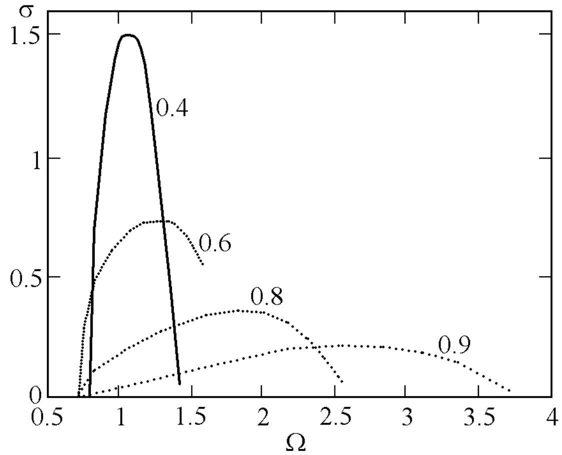


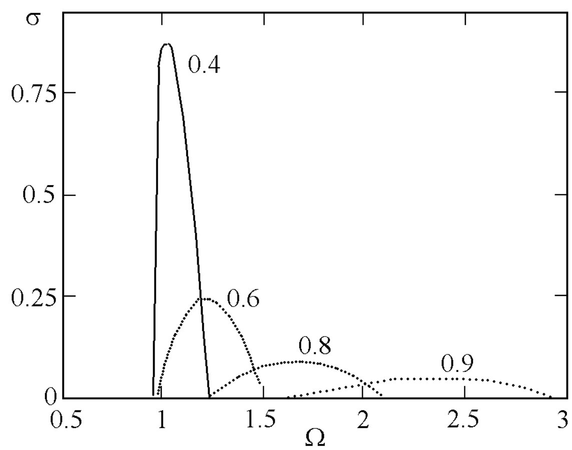


**Figure 1.** Spectral density of the radiation in units $p'^2_0\omega'^4_0/3c^3$ for the case of the plasma at (a) $\Omega_p = 0.7$ and (b) $\Omega_p = 0.95$. Values of the oscillator motion velocity $\beta$ are shown at the curves.

## 2. Case of Cold Plasma

[5] Let an electric dipole in the "proper" reference system is characterized by the dipole moment density $\mathbf{P}' = \mathbf{p}'\delta^3(x', y', z')$, where $\mathbf{p}' = p'_0\mathbf{e}'_z \exp(-i\omega_0 t')$ and $\omega'_0$ is the source oscillation frequency in this reference system (the values related to the source "proper" reference system are marked by dashes). We will consider the dipole moving along the direction of its dipole moment with a velocity of $\mathbf{v} = v\mathbf{e}_z$ ($v > 0$). In this case, in the "laboratory" reference system, the source also has only an electric dipole moment equal to $\mathbf{p} = \mathbf{p}'\sqrt{1-\beta^2}$, where $\beta = v/c$. The surrounding medium taken to be homogeneous, isotropic, and nonabsorbing has permittivity ($\varepsilon$) and permeability ($\mu$) which depend on the wave frequency but are independent of the wave vector.

[6] In this paper we will analyze the radiation power $\Sigma$ and its spectral density $\sigma(\omega)$ averaged over a period. The general expression for $\Sigma$ [*Ginzburg and Frank*, 1947b] may be written in the following form:

$$\Sigma = \int\limits_{\omega>0} \sigma(\omega) d\omega \qquad \sigma(\omega) = \tilde{\sigma}(\omega) + \tilde{\sigma}(-\omega) \tag{1}$$

$$\tilde{\sigma}(\omega) = \frac{p'^2_0(1-\beta^2)}{4c^2 v}\mu|\omega|^3 \times$$

$$\left[1 - \left(\frac{\omega-\omega_0}{\omega n\beta}\right)^2\right] 1[(\omega n\beta)^2 - (\omega-\omega_0)^2] \tag{2}$$

where $n^2(\omega) = \varepsilon(\omega)\mu(\omega)$, $\omega_0 = \omega'_0\sqrt{1-\beta^2}$, and $1(\xi)$ is the unit function of Heaviside:

$$1(\xi) = \begin{cases} 1 & \text{if} \quad \xi > 0 \\ 0 & \text{if} \quad \xi < 0 \end{cases}$$

One can see that the $\sigma(\omega)$ value presenting the spectral density of the radiation energy differs from zero within the frequency range determined by the inequality

$$\omega^2\beta^2\varepsilon(\omega)\mu(\omega) > (\omega-\omega_0)^2 \tag{3}$$

Further analysis of the energetic characteristics depends on the choice of the medium model. In publications [see, e.g., *Frank*, 1942; *Ginzburg and Frank*, 1947b; *Tyukhtin*, 2004a] the simplest case of a medium without dispersion is considered in a most detailed way. Not discussing this problem we come to the analysis of the energy loss in cold plasma characterized by the permittivity $\varepsilon = 1 - \omega_p^2/\omega^2$ (where $\omega_p$ is the plasma frequency) and permeability $\mu = 1$. In this situation the solution of inequality (3) determining the frequency range of the radiated waves takes the form

$$\omega_1 < \omega < \omega_2$$

$$\omega_{1,2} = \frac{\omega'_0 \mp \beta\sqrt{\omega'^2_0 - \omega_p^2}}{\sqrt{1-\beta^2}} \tag{4}$$

Radiation occurs only in the case when the values $\omega_{1,2}$ are real, i.e., $\omega'_0 > \omega_p$ (it is worth emphasizing that the source frequency in the laboratory frame of reference equal to $\omega_0 = \omega'_0\sqrt{1-\beta^2}$ may be even lower than the plasma frequency). One can easily see that the width of the radiation spectrum increases with an increase of the source motion velocity and decreases with an increase of the plasma frequency.

[7] The spectral power of the radiation has the form

$$\sigma(\omega) = \frac{p'^2_0}{4c^3}\frac{(1-\beta^2)^2}{\beta^3}\frac{\omega'_0\omega^3(\omega-\omega_1)(\omega_2-\omega)}{\omega^2-\omega_p^2} \times$$

$$1(\omega-\omega_1)1(\omega_2-\omega) \tag{5}$$

Figures 1a and 1b show the dependencies of the spectral power of the radiation (in the units $p'^2_0\omega'^4_0/3c^3$) on the dimensionless frequency $\Omega \equiv \omega/\omega'_0$ for various value of $\beta$ and $\Omega_p \equiv \omega_p/\omega'_0$. It is worth noting that at not too large values of the plasma frequency, the character of spectral distributions is similar to the one taking place in vacuum. However, if $\Omega_p \approx 1$ (Figure 1b), the frequency distributions of the power take an interesting peculiarity: at rather high velocity, the radiation spectrum is entirely located at $\Omega > 1$, that is, in the region of the frequencies exceeding the proper frequency of the oscillator $\omega'_0$.

[8] The total radiation power of an electric dipole obtained after substitution of (5) into (1) and calculation of the corresponding integral is written in the following form:

$$\Sigma = \frac{p'^2_0\omega'^4_0}{4c^3\beta^3}\left\{2\beta\sqrt{1-\Omega_p^2}\left[\Omega_p^2+\frac{\beta^2}{3}(2-5\Omega_p^2)\right]-\right.$$

$$\frac{\Omega_p^2}{2}(1-\beta^2)\left[(\Omega_p+\sqrt{1-\beta^2})^2\ln\frac{\Omega_2+\Omega_p}{\Omega_1+\Omega_p}+\right.$$

$$\left.\left.(\Omega_p-\sqrt{1-\beta^2})^2\ln\frac{\Omega_2-\Omega_p}{\Omega_1-\Omega_p}\right]\right\} \qquad (6)$$

where $\Omega_{1,2} = \omega_{1,2}/\omega'_0$. One can show that this expression is a monotonously decreasing function of both the velocity of the dipole motion and plasma frequency. At low velocities the function coincides with the accuracy up to the value of the order of $\beta^2$ with the radiation power of a motionless source:

$$\Sigma \approx \frac{p'^2_0\omega'^4_0}{3c^3}(1-\Omega_p^2)^{1/2}$$

In the ultrarelativistic regime when $1-\beta^2 \ll 1$, one can obtain

$$\Sigma \approx \frac{p'^2_0\omega'^4_0}{3c^3}(1-\Omega_p^2)^{3/2}$$

[9] Figure 2 shows the dependencies of the radiation power of an electric dipole on the velocity of its motion at several values of the plasma frequency. One can see that this dependency is insignificant if the plasma frequency $\omega_p$ is not too close to the proper frequency of the source $\omega'_0$. It is worth emphasizing that, in spite of this, the radiation spectrum undergoes quite significant reconstruction at changes in the velocity, this fact being mentioned above.

[10] Concluding this section, we make some notes concerning radiation of the moving longitudinal magnetic dipole in cold plasma (more details on this problem are given by *Tyukhtin* [2004a]). One can show that the total power of radiation of a magnetic oscillator is

$$\Sigma_m = \frac{m'^2_0\omega'^4_0}{3c^3}(1-\Omega_p^2)^{3/2}$$

where $m'_0$ is the amplitude value of the magnetic dipole moment in the reference system of the source. Thus $\Sigma_m$ does not at all depend on the source motion velocity, in spite of the significant dependence of the spectral composition of the radiation. At equal dipole moments and proper frequencies, an electric dipole is more effective emitter than a magnetic one, because $\Sigma_m \le \Sigma$ (the equality of the powers of this two sources is reached only in the limit $\beta \to 1$).

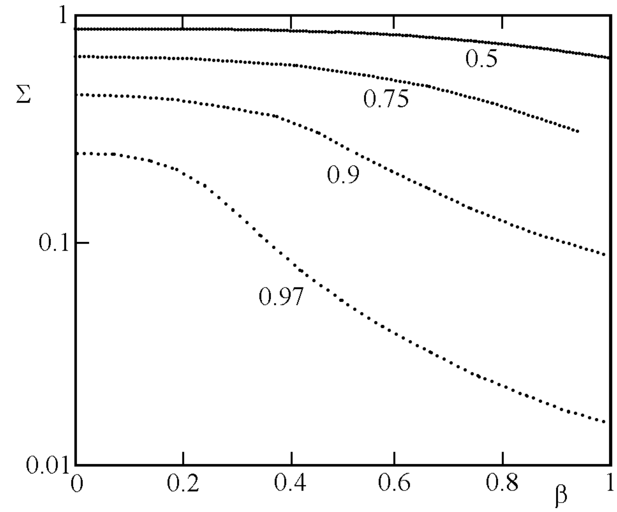


**Figure 2.** Dependence of the radiated power of an electric dipole in units $p'^2_0\omega'^4_0/3c^3$ on the velocity in the case of plasma. The values of the plasma frequency $\Omega_p$ are indicated at each curve.

## 3. Case of Resonant Medium

[11] Now we consider such case when an electric dipole is moving in the nonmagnetic and nonabsorbing medium having the dispersion of a resonant type. We have for the medium with one resonant frequency:

$$\varepsilon = 1 + \frac{\omega_p^2}{\omega_r^2-\omega^2} = \frac{\omega_r^2\varepsilon_0-\omega^2}{\omega_r^2-\omega^2}, \qquad \mu = 1 \qquad (7)$$

where $\omega_r$ and $\omega_p$ are the resonant and plasma frequencies, respectively, and $\varepsilon_0 = 1+\omega_p^2/\omega_r^2$ is the permittivity of the medium relative to the static field. It is worth noting that for such medium, a detailed study of the radiation even in the simplest case of a moving point charge was performed only in the recent years. In particular, *Afanasiev and Kartavenko* [1998] and *Afanasiev et al.* [1999] analyzed the radiation of a charge in an infinite resonantly dispersive dielectric, and *Tyukhtin* [2004b, 2005] considered the radiation in a waveguide filled in by a dielectric.

[12] Substituting (7) into (2), we obtain

$$\tilde{\sigma}(\omega) = \frac{p'^2_0(1-\beta^2)}{4v^3}\frac{|\omega|f(\omega)}{\omega_r^2+\omega_p^2-\omega^2} \qquad (8)$$

where

$$f(\omega) = (1-\beta^2)\omega^4 - 2\omega_0\omega^3 +$$

$$[\omega_0^2-\omega_r^2(1-\beta^2)+\omega_p^2\beta^2]\omega^2 + 2\omega_0\omega_r^2\omega - \omega_0^2\omega_r^2 \qquad (9)$$

Condition (3) determining the range of the emitted frequencies is reduced to the following requirements:

$$f(\omega) > 0 \ \text{ at } \omega^2 \le \omega_r^2$$

$$f(\omega) < 0 \text{ at } \omega^2 \geq \omega_r^2 + \omega_p^2 \tag{10}$$

(in the frequency region $\omega_r^2 < \omega^2 < \omega_r^2 + \omega_p^2$, there can be no radiation at all, because condition (3) is not fulfilled due to negative value of $\varepsilon$).

[13] One can obtain relatively simple formulae for the boundary frequencies and radiation energies at some limitations on the problem parameters. We present below only the estimates for boundary frequencies. We assume first that the resonant frequency of the medium is much less than the oscillator frequency in the laboratory frame of reference: $\omega_r \ll \omega_0 = \omega_0'\sqrt{1-\beta^2}$. Then it is easy to show that there exist two frequency ranges in the radiation spectrum. The first one is determined by the inequalities $\omega_* < \omega < \omega_r$, where $\omega_* \approx \omega_r(1+\beta^2\omega_p^2\omega_0^{-2})^{-1/2}$. This frequency range may be called a "resonant" one because it is located in the vicinity of the resonance frequency $\omega_r$. The second emitted frequency range is determined by the inequality $\omega_1 < \omega < \omega_2$, where $\omega_{1,2} \approx (\omega_0' \mp \beta\sqrt{\omega_0'^2 - \omega_p^2})/\sqrt{1-\beta^2}$. This range may be called a "proper" one because at relatively small velocities it includes the oscillator frequency $\omega_0'$ (however, it should be borne in mind that at sufficiently high values of $\beta$, the lower boundary of this range becomes higher than $\omega_0'$). We emphasize that this frequency range in the radiation spectrum exists only under condition $\omega_0' > \omega_p$. It is worth also noting that the "resonant" and "proper" frequency ranges at the condition $\omega_r \ll \omega$, as a rule, are located rather far from each other. One can show that the "resonant" radiation is much weaker than the "proper" radiation (if the latter does exist). We emphasize that in the case $\omega_0 < \omega_p$, the "proper" radiation disappears, whereas the "resonant" radiation takes place at any relation between the plasma frequency and proper frequency of the source.

[14] If $\omega_r \gg \omega_0$, two principally different possibilities may be realized. The first one takes place at $\beta\sqrt{\varepsilon_0} < 1$ when the dipole motion velocity $v$ is less than the phase velocity of the low-frequency radiation ($c/\sqrt{\varepsilon_0}$). In this case there exist both the "proper" range of the emitted frequencies and the "resonant" range adjacent to the frequency $\omega_r$. For the "resonant" radiation under the additional condition $\omega_0\sqrt{1-\beta^2} \ll \omega_r\sqrt{1-\beta^2\varepsilon_0}$, we obtain the frequency range $\omega_* < \omega < \omega_r$, where $\omega_* \approx \omega_r\sqrt{(1-\beta^2\varepsilon_0)/(1-\beta^2)}$. For the "proper" radiation under the additional condition $\omega_0 \ll \omega_r(1-\beta\sqrt{\varepsilon_0})$, we obtain the frequency range $\omega_1 < \omega < \omega_2$, where $\omega_{1,2} \approx \omega_0/(1 \pm \beta\sqrt{\varepsilon_0})$. It is worth noting that in these conditions both the "resonant" and "proper" radiations may prevail.

[15] In the case when $\omega_0 \ll \omega_r$ but $\beta\sqrt{\varepsilon_0} > 1$, the radiation spectrum contains only one frequency range $\omega_* < \omega < \omega_r$, where $\omega_* \approx \omega_0/(1+\beta\sqrt{\varepsilon_0})$. One can show that in this situation, the total radiation power depends very weakly on the oscillator frequency.

[16] Figures 3a, 3b, 3c, and 3d show the spectral density of the radiation energy as a function of the dimensionless frequency $\Omega = \omega/\omega_0'$ at various values of the dimensionless resonant ($\Omega_r = \omega_r/\omega_0'$) and plasma ($\Omega_p = \omega_p/\omega_0'$) frequencies and the source motion velocity. Figure 3a shows a typical picture for the situation when both the resonant and plasma frequencies are less than the oscillator frequency: $\omega_r < \omega_0'$ and $\omega_p < \omega_0'$. In this case, there are two frequency ranges: the "proper" radiation is dominating and the "resonant" one is insignificant. Figure 3b illustrates the case when $\omega_r < \omega_0'$ and $\omega_p > \omega_0'$. In this case there is only one ("resonant") range of radiation frequencies.

[17] Figure 3c is typical for the case when $\omega_r > \omega_0'$ and $\omega_p < \omega_0'$. In this case three possibilities may be realized. If the dipole motion velocity is less than some value $\beta_*$ ($\beta_* \approx 0.433$ for the values of the parameters used in Figure 3c), there are both the "resonant" and "proper" (relatively low-frequency) ranges. If $\beta_* < \beta < \beta_{**}$ (in our case $\beta_{**} \approx 0.82$), there is only one frequency range including the oscillator frequency and adjacent to the resonant frequency. If the oscillator velocity is high enough ($\beta > \beta_{**}$), then (besides this range) there is one more range lying above the resonance frequency. Figure 3d illustrates the case when $\omega_r > \omega_0'$. and $\omega_p > \omega_0'$. In this case two possibilities can be realized: either there are two frequency ranges (if the velocity is low enough), or there is only one range.

[18] Figures 4a and 4b show the dependence of the total power of the dipole radiation on the velocity of its motion at various resonant and plasma frequencies. Figure 4a corresponds to the case when the resonant frequency is lower than the oscillator frequency ($\Omega_r < 1$). If the plasma frequency is also lower than the oscillator frequency ($\Omega_p < 1$), there are both the "proper" and "resonant" radiations, the former one prevailing. In this situation, the dependence of the radiation power on the velocity is insignificant. If $\Omega_p > 1$, only relatively weak "resonant" radiation is generated. It has fairly well pronounced dependence on the velocity with a maximum at some of its value. Figure 4b corresponds to the case when the resonant frequency exceeds the oscillator frequency. In this case the dependence of the power on the velocity has a maximum, its value increasing with an increase of the plasma frequency.

[19] Concluding this section, we emphasize that the comparison of the obtained results with the results for the cases of nondispersive medium and cold plasma shows that the presence of the resonant dispersion leads to different, much more complicated, regularities characterizing the radiation of a moving oscillator.

## 4. Case of Moving Medium

[20] In this section we consider such situation, when not only the source is moving but the surrounding medium is moving as well. The study of the case of a point charge in these conditions [*Bolotovskiy and Stolyarov*, 1983] revealed, in particular, the presence of the effect of the reversal of the energy loss sign. So it is quite interesting to study radiation of oscillating sources. This problem was partly considered by *Garibyan and Kostanyan* [1971]; however, no analysis of energetic regularities was performed.

[21] We will consider (as it has been done above) a source having only the electric dipole moment $\mathbf{p}' = p_0'\mathbf{e}_z' \exp(-i\omega_0' t')$ in the "proper" reference system (PRS) and moving with a constant velocity $\mathbf{v} = v\mathbf{e}_z$ relative to the "laboratory" reference system (LRS). Unlike in sections 2 and 3, we will now assume that the medium moves with a velocity $\mathbf{u} = u\mathbf{e}_z$ rel-

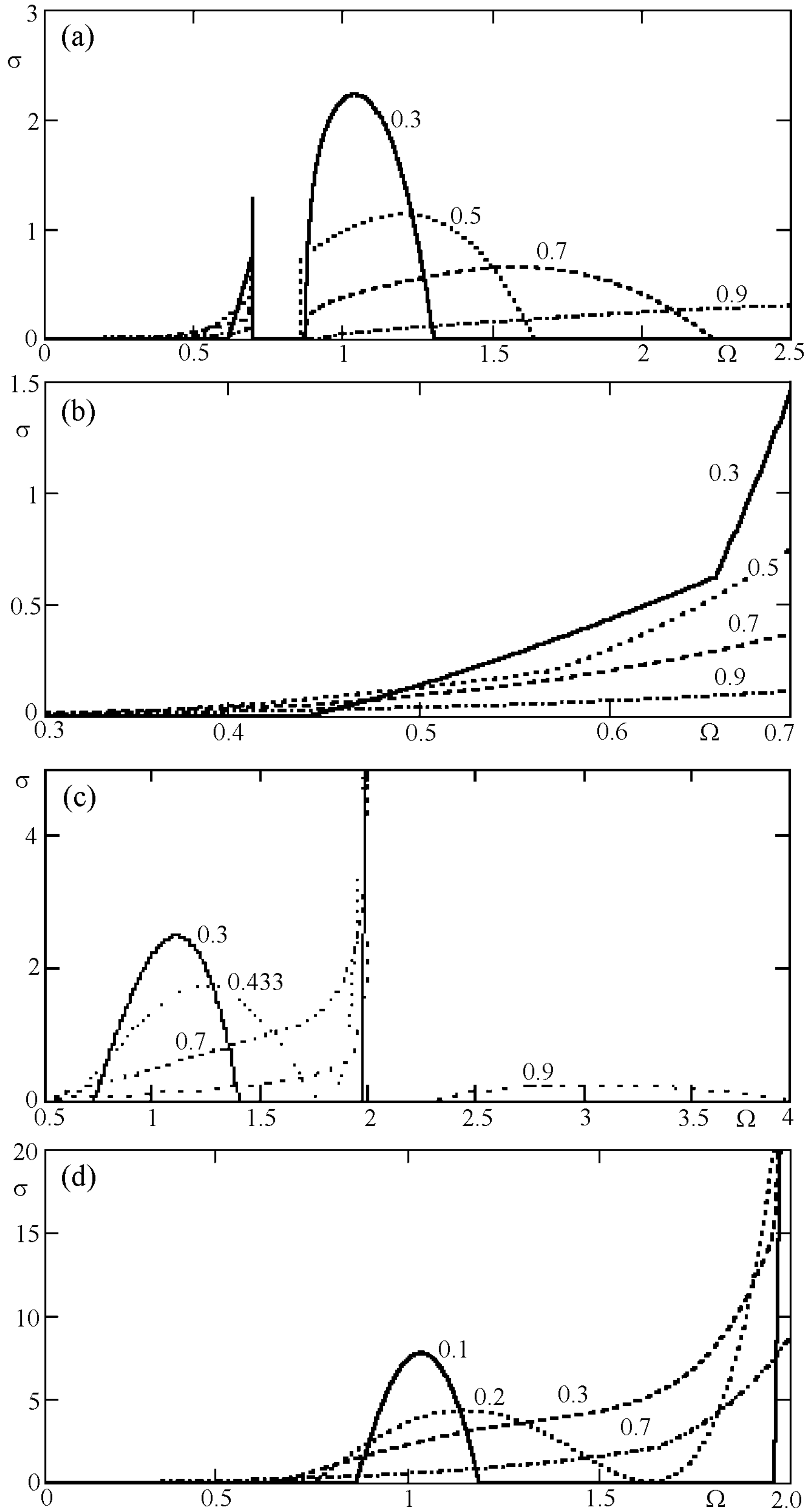


**Figure 3.** Spectral density of the radiated power (in units $p_0'^2\omega_0'^4/3c^3$) for the case of the resonantly dispersive medium: (a) $\Omega_r = 0.7$ and $\Omega_p = 0.5$; (b) $\Omega_r = 0.7$ and $\Omega_p = 2$; (c) $\Omega_r = 2$ and $\Omega_p = 0.5$; (d) $\Omega_r = 2$ and $\Omega_p = 2$. The values of the oscillator motion velocity are indicated at each curve.

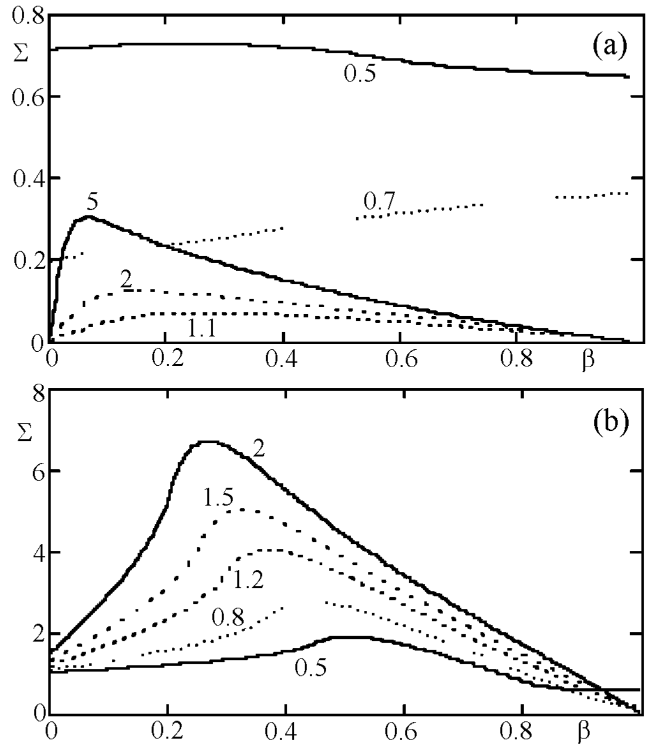


**Figure 4.** Radiated power (in units $p_0'^2\omega_0'^4/3c^3$) for the case of the resonantly dispersive medium as a function of the dipole motion velocity $\beta$: (a) $\Omega_r = 0.7$ and (b) $\Omega_r = 2$. The values of the plasma frequency $\Omega_p$ are indicated at each curve.

ative to LRS. For the sake of definiteness, we take $u > 0$, whereas the sign of $v$ may be arbitrary. In this section we consider the case of the medium with the refraction index $n = \sqrt{\varepsilon\mu} > 1$ which may be considered almost constant within a wide frequency range.

[22] The procedure of determination of the field in this problem may be different. On one hand, one is able to solve directly equations for this or that potential (for example, for the Hertz vector) in the moving medium using the Fourier method and taking into account the causality principle or the Mandelstam radiation condition. On the other hand, one can use the known spatial–time Green function [*Bolotovskiy and Stolyarov*, 1983]. Certainly, all the ways give the same final result. Below we describe the former way.

[23] From the equations for electromagnetic field in a moving medium [*Bolotovskiy and Stolyarov*, 1983], there follows the following expression for the Fourier image of the Hertz vector:

$$\mathbf{\Pi}_\omega = 4\pi \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \mathbf{P}_{\omega,\mathbf{k}} \times$$

$$\frac{\mu \exp(i\mathbf{k}_\rho\boldsymbol{\rho} + ik_z z)d^2\mathbf{k}_\rho dk_z}{k_\rho^2 + k_z^2 - \omega^2 c^{-2} - (n^2-1)\gamma_u^2 c^{-2}(\omega - k_z u)^2} \qquad (11)$$

where $\gamma_u = (1-\beta_u^2)^{-1/2}$, $\beta_u = u/c$, $\rho = x\mathbf{e}_x + y\mathbf{e}_y$, $\mathbf{k}_\rho = k_x\mathbf{e}_x + k_y\mathbf{e}_y$, and $\mathbf{P}_{\omega,\mathbf{k}} = \mathbf{e}_z p(2\pi)^{-3}\delta(\omega - \omega_0 - vk_z)$. It is reasonably to calculate first the integral of $\exp(i\mathbf{k}_\rho\boldsymbol{\rho})$ over the angle between $\rho$ and $\mathbf{k}_\rho$ (it is equal to $2\pi J_0(k_\rho\rho)$, where $J_0(x)$ is the Bessel function), and then to reduce the integral along the semiaxis ($0 < k_\rho < \infty$) to the integral along the entire real axis ($-\infty < k_\rho < \infty$). The latter integral will contain the Hankel function $H_0^{(1)}(k_\rho\rho)$, and the integrating contour going along the upper shore of its cut. Determining the rule of the going round the poles of the integrand on the $k_\rho$ plane, one should use the Mandelstam radiation principle requiring the group velocity of the propagating waves $\mathbf{v}_g$ to be directed from the $z$ axis [*Bolotovskiy and Stolyarov*, 1972]. Using the known properties of the group velocity of waves in a moving medium [*Bolotovskiy and Stolyarov*, 1976], one can easily find which of the poles on the $k_\rho$ plane should be gone round from below (the condition $v_{g\rho} > 0$ should be fulfilled for it) and which one should be gone round from above. After determination of the positions of the poles, the integral over $k_\rho$ is found by closing the contour into the upper semiplane. The integral over $k_z$ is taken easily due to the fact that the Fourier image $\mathbf{P}_{\omega,\mathbf{k}}$ contains a delta function. As a result, we have

$$\Pi_\omega = \mathbf{e}_z \frac{ip_0}{2|v|}\mu H_0^{(1)}(s\rho)\exp\left(i\frac{\omega-\omega_0}{v}z\right) \qquad (12)$$

The value $s$ squared standing in the argument of the Hankel function is

$$s^2 = v^{-2}\gamma_u^2(n^2-\beta_u^2)(\beta_v-\beta_{v1})(\beta_v-\beta_{v2})\times$$

$$(\omega-\omega_1)(\omega-\omega_2) =$$

$$c^{-2}\beta_0^{-2}(n^2\beta_0^2-1)(\tilde{\omega}-\tilde{\omega}_1)(\tilde{\omega}-\tilde{\omega}_2) \qquad (13)$$

where

$$\omega_{1,2} = \omega_0'\sqrt{1-\beta_v^2}\frac{\beta_{v1,2}}{\beta_{v1,2}-\beta_v}$$

$$\beta_{v1,2} = \mp\frac{1\mp n\beta_u}{n\mp\beta_u}$$

$$\beta_v = \frac{v}{c}$$

$$\tilde{\omega} = \gamma_u\omega(1-\frac{u}{v})$$

$$\tilde{\omega}_{1,2} = \frac{\omega_0'\sqrt{1-\beta_0^2}}{1\pm n\beta_0}$$

$$\beta_0 = (\beta_v-\beta_u)(1-\beta_v\beta_u)^{-1} \qquad (14)$$

Here the variable $\tilde{\omega}$ has the sense of the frequency in the rest system of the medium, and $\beta_0$ represents the dipole motion velocity in this system.

[24] We see that the $s$ value may be either real, or imaginary. In the latter case, its imaginary part is positive ($s = i|s|$) and that provides exponential decrease of the generated "inhomogeneous waves". In the $s^2 > 0$ case, the physically correct sign of $s$ is obtained as a result of the application of the Mandelstam radiation principle as it has been mentioned above. It is determined by the following rules:

[25] 1. If $\beta_{v1} < \beta_v < \beta_{v2}$ (i.e., $n^2\beta_0^2 < 1$), the value $s$ is real only in the limited region of frequencies $\omega_{\min} < \omega < \omega_{\max}$, where $\omega_{\min} = \min\{\omega_1, \omega_2\}$, and $\omega_{\max} = \max\{\omega_1, \omega_2\}$. In this case $s > 0$.

[26] 2. If $\beta_v < \beta_{v1}$ or $\beta_v > \beta_{v2}$ (i.e., $n^2\beta_0^2 > 1$), the value of $s$ is real in two semilimited regions: at $\omega < \omega_{\min}$ and $\omega > \omega_{\max}$. In this case,

$$\operatorname{sgn} s = \operatorname{sgn}\left[(\beta_v - \beta_u)\beta_v\right] \begin{cases} -1 & \text{for} \quad \omega < \omega_{\min} \\ \phantom{-}1 & \text{for} \quad \omega > \omega_{\max} \end{cases}$$

It is worth emphasizing that the inequalities $\beta_{v1} < \beta_v < \beta_{v2}$ are equivalent to the requirement $n^2\beta_0^2 < 1$ which means that the dipole moves relative the medium with the velocity lower than the phase speed of light in the medium. Therefore the given case may be called a regime of "subluminal relative motion" (SubRM). The inequalities $\beta_v < \beta_{v1}$ or $\beta_v > \beta_{v2}$ are equivalent to the requirement $n^2\beta_0^2 > 1$ and therefore this case may be called a regime of "superluminal relative motion" (SupRM). As in the case of a motionless medium, in the SubRM regime, the range of the emitted frequencies (i.e., the region of the real values of $s$) is finite, whereas in the SupRM regime it is infinite.

[27] Components of the electromagnetic field in the moving medium are expressed via the Hertz vector according to the known formulae [*Bolotovskiy and Stolyarov*, 1983] (we would not write them down here). The calculation of the spent to radiation source power averaged over the period we perform using the Pointing vector integration over the surface surrounding the source. If one, as usually, takes as such a surface an infinite cylinder surface the axis of which coincides with the source motion trajectory, one obtains the following expression for the averaged power

$$\Sigma = -\frac{c}{4}\int_{-\infty}^{\infty} E_z H_\varphi^* dz \tag{15}$$

One can transform this integral identically to a more simple form using methods used in the classical theory of radiation of moving sources in the motionless medium [*Frank*, 1981]. Omitting all intermediate calculations, we present here the final result of this procedure:

$$\Sigma = \int_{\omega>0} \sigma(\omega)d\omega$$

$$\sigma(\omega) = \tilde{\sigma}(\omega) + \tilde{\sigma}(-\omega) \tag{16}$$

$$\tilde{\sigma}(\omega) = \frac{p_0'^2(1-\beta_v^2)}{4|v|\varepsilon}\omega s^2(\omega)\operatorname{sgn}(s)1(s^2) \tag{17}$$

The presence of the unit Heaviside function $1(s^2)$ shows that (as in a motionless medium) the integration is performed only over the part of the frequency axis where the value $s(\omega)$ is real (which is quite natural because only in this frequency range there are propagating waves).

[28] It is worth noting that integral (16) is convergent only in the case of a limited radiation spectrum, that is, at $\beta_{v1} < \beta_v < \beta_{v2}$. If the spectrum is unlimited, integral (16) is divergent in the case of the medium without dispersion. Here we have an analog of the energetic "paradox" known in the Vavilov-Cerenkov radiation theory [*Bolotovskiy*, 1957; *Frank*, 1981; *Ginzburg*, 1987, 2002; *Zrelov*, 1968]: the source moving with a superluminal velocity in a motionless nondispersive medium should loose infinite energy in a unit of time. In our case such situation arises in the SupRM motion, i.e., at either $\beta_v < \beta_{v1}$ or $\beta_v > \beta_{v2}$. It is known that this paradox may be resolved, for example, at taking into account the frequency dispersion of the medium which inevitably leads to limitations of the radiation spectrum.

[29] Calculating integral (15) in the $n^2\beta_0^2 < 1$ case, we obtain

$$\Sigma = \frac{\omega_0'^4 p_0'^2 n\mu}{3c^3}\frac{(1-\beta_0^2)^3}{(1-n^2\beta_0^2)^3}\frac{1+n^2\beta_0\beta_u}{1+\beta_0\beta_u} \tag{18}$$

In particular cases of motionless medium and motionless source, expression (18) is reduced to the known results: at $\beta_u = 0$ we have [*Ginzburg and Frank,* 1947b]

$$\Sigma = \frac{\omega_0'^4 p_0'^2 n\mu}{3c^3}\frac{(1-\beta_v^2)^3}{(1-n^2\beta_v^2)^3}$$

and at $\beta_v = 0$ we have [*Daly et al.*, 1965; *Doil'nitsyna and Tyukhtin,* 2003, 2004]

$$\Sigma = \frac{\omega_0'^4 p_0'^2 n\mu}{3c^3}\frac{(1-\beta_u^2)^2}{(1-n^2\beta_u^2)^2}$$

The most interesting feature of expression (18) is the fact that it changes sign at some parameters of the problem. Under the condition $n^2\beta_0\beta_u > -1$ which is equivalent to inequality

$$\beta_v > \beta_v^* \equiv \frac{n^2\beta_u^2 - 1}{\beta_u(n^2-1)}$$

the radiation power is positive, whereas under the condition $n^2\beta_0\beta_u < -1$ (i.e., at $\beta_v < \beta_v^*$) it becomes negative. The latter means that if the source lags considerably behind the medium stream, it obtains energy from the energy of the moving medium. It should be noted that in the considered SubRM regime such effect is possible only for the "superluminal" flow of the medium when $n\beta_u > 1$ (because only in this case the inequality $\beta_{v1} < \beta_v^* < \beta_{v2}$ is fulfilled). For its realization, the velocity of the source motion should be considerably less than the medium motion velocity, because $\beta_v^* < \beta_u$). Certainly, this effect will take place also in the SupRM regime; however, in this case the radiation power in the model of nondispersive medium appears to be infinite (as it has been already noted, this "paradox" may be resolved by taking into account the dispersion).

[30] The effect of the sign reversal of the energy wave losses was noted earlier for the case of a charge moving in the moving medium [see, e.g., *Bolotovskiy and Stolyarov*, 1983]. However, in this case the radiation field presents only in the SupRM regime and, respectively, only in such regime the effect of the sign reversal of energy wave losses is possible. To be exact, the effect is realized under the condition $\beta_v < \beta_{v1}$, whereas for the oscillator this effect takes place under the condition $\beta_v < \beta_v^*$. Since $\beta_v^* > \beta_{v1}$, it is evident that this phenomenon for an oscillator occurs at smaller difference in velocities $\beta_u - \beta_v$ than for a charge.

## 5. Conclusions

[31] In this paper, the radiation power of a moving oscillating electric dipole in some media was analyzed. It was assumed that its dipole moment is oriented along the motion velocity. The main results of the paper are the following.

[32] In the case when the surrounding medium is cold plasma, it is shown that the radiation occurs only when the proper frequency of the oscillator $\omega_0'$ exceeds the plasma frequency. The radiation spectra are similar to those in the case of vacuum if the plasma frequency is not too close to the proper frequency of the oscillator. If the both frequencies slightly differ from each other, at a high enough velocity the radiation spectrum lies completely in the region above $\omega_0'$. The radiation power is a monotonously decreasing function of the velocity and plasma frequency.

[33] In the case when the medium has the dispersion of a resonant type, at some parameters of the problem the radiation spectrum consists of two separated frequency ranges, whereas at other parameters this two ranges are united into one range. The dependence of the radiation power on the source motion velocity is different at different relations between the resonance frequency, plasma frequency, and oscillator frequency. The comparison of the obtained results with the results for the corresponding problems in the case of nondispersive medium and cold plasma, shows that the presence of the resonant dispersion leads to different, much more complicated regularities characterizing the radiation of a moving oscillator.

[34] The expression for the radiation power is derived for the case when not only the dipole moves, but the surrounding it nondispersive medium moves as well, the motion velocities of the source and medium being parallel or antiparallel. It is noted that, in particular, at some parameters of the problem, the effect of the reversal of the wave losses sign takes place, that is the radiating power becomes negative. Unlike in the case of a moving charge, for an oscillator this effect can take place even in the regime of the "subluminal relative motion".

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