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Radiation hydrodynamics of the stratified solar plasma

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Abstract. Helioseismology makes it possible to determine the conditions in the solar interior using information from the acoustic waves (p modes) propagating through the Sun and being detected by remote sensing techniques on its surface. The p modes seem to be stochastically excited and mainly damped by radiative losses. Besides, *Staude et al.* [1994] mentioned that to analyze the velocity and radiation intensity oscillations obtained by the CORONAS and SOHO satellites, wave models for stratified plasmas have to be taken into account. Therefore, in reality, one has to consider magnetoacoustic gravity waves. Thus an attempt is made here to develop the theory of radiation hydrodynamics for the stratified solar plasma considering the radiation transport in Eddington approximation. The temperature distribution for stratified plasma is assumed to be nonuniform in the same way as it has been done by *Babaev et al.* [1995]. In the momentum balance, the altitude-dependent radiation pressure is taken into account. Comparing with other works, here the momentum and energy balances for particles and radiation are studied. Similar to the works for stratified plasmas neglecting nonadiabatic effects, a dispersion relation is found introducing the divergence of the plasma velocity as a new parameter.

1. Introduction

Studying global solar oscillations, properties in the velocity and radiation intensity fluctuations of the acoustic-type modes (called p modes) were found, which provides a reason to conclude that the waves behave nonadiabatically [e.g., *Deubner*, 1991]. The most probable reasons for this nonadiabaticity are interactions between the acoustic waves and the processes of radiation and heat transport as well as turbulence. First theoretical investigations of the visibility of the luminosity variations of the Sun correlated with p modes were performed by *Toutain and Gouttebroze* [1988, 1993].

The interpretation of the wave observations was mostly limited by adiabatic models. In general, the nonadiabatic models are yet restricted to the case of gray, homogeneous atmospheres. Thus the further development of the theory of nongray and nonhomogeneous atmospheres stratified in the gravity field and structured by the magnetic field is needed to understand the more and more detailed experimental data.

Thus *Ibánez and Plachco* [1989] introduced in the basic radiation magnetohydrodynamic equations of ideal gases [*Mihalas and Mihalas*, 1984] the electron heat conductivity. Already *Souffrin* [1972] obtained a dispersion equation of nonadiabatic waves for an isothermal atmosphere using the Newton approximation for the radiation relaxation. *Zhugzhda* [1983] found an analytical solution for nonadiabatic waves in an isothermal atmosphere with density-dependent radiation transport. Further, in 1991, he presented a general solution for the fundamental mode of any atmosphere at all.

However, in all the publications mentioned, the magnetic field was neglected. The influence of the magnetic field in an isothermal stratified atmosphere was first considered by *Babaev et al.* [1995], who used the Newton approximation for the radiation loss term.

Staude et al. [1994] studied nonadiabatic hydrodynamic waves, for example, the 5-min oscillations of the solar photosphere and chromosphere, in a homogeneous, nongray, radiating and thermally conducting atmosphere under the condition of thermal equilibrium. It was found that the heat conductivity by particle collisions only (neglecting effective

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wave-particle collisions) seems to be less important than the heat conductivity due to the radiation transport.

In general, the numerical modeling of the nonadiabatic waves in radiating atmospheres showed that also the further development of analytical solutions is of importance, on the one side, to define the boundary conditions for the numerical calculations [*Dzhalilov et al.*, 1992] and, on the other side, to guarantee a better analysis of the numerical results. Thus an analytical study of the radiation hydrodynamics of a stratified atmosphere is presented here.

2. Eddington Approximation of Radiation Magnetohydrodynamics

To describe low-frequency phenomena in a fluid plasma system with radiation transport being influenced by a gravity force, one can use radiation hydrodynamics. This means that the continuity equation

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{u}) = 0 \tag{1}$$

the momentum balance

$$\rho \frac{d\mathbf{u}}{dt} = -\nabla(p + P_R) + \rho \mathbf{g} \tag{2}$$

and the energy balance of a nonadiabatic plasma system

$$\frac{dp}{dt} - c_s^2 \frac{d\rho}{dt} = (1-\gamma)L = 4\pi(\gamma - 1)\kappa(J - S) \tag{3}$$

should be considered: $d/dt = (\partial/\partial t + \mathbf{u} \cdot \nabla)$, where ρ is the mass density, p is the scalar pressure, $\rho\mathbf{u}$ is the momentum density, $\mathbf{g}$ is the gravitational acceleration, and γ is the adiabatic coefficient of the plasma. S is the radiation source function, J represents the mean intensity of the scattered radiation field, L describes the radiation loss function, and κ designates the opacity. Taking into account that the square of the sound velocity is equal to $c_s^2 = \gamma p/\rho$, one can also present (5) in the form

$$\frac{\rho^\gamma}{\gamma - 1} \frac{d}{dt}\left(\frac{p}{\rho^\gamma}\right) = -L \tag{4}$$

Further, it is assumed that the thermodynamic equation of an ideal gas is applicable:

$$p = \frac{R\rho T}{\tilde{\mu}} \tag{5}$$

where $\tilde{\mu}$ is the mean molecular weight. The radiation pressure P_R is given by *Mihalas and Mihalas* [1984]

$$P_R = \frac{4\pi J}{3c} \tag{6}$$

The source function S of the electromagnetic radiation is assumed to be equal to the Planck function

$$S = \frac{\sigma T^4}{\pi} \tag{7}$$

where $\sigma = (2\,\pi^5\,k_B^4)\,/\,(15\,h^3\,c^2) = (5.6697 \pm 0.0029) \times 10^{-8}$ W m^2 K^{-4} is the Stefan-Boltzmann constant.

The momentum and energy equations for the radiation field are used in the Eddington approximation [*Mihalas and Mihalas*, 1984]

$$\frac{1}{c}\frac{d\mathbf{H}}{dt} + \frac{1}{3}\nabla J = -\kappa \mathbf{H} \tag{8}$$

$$\frac{1}{c}\frac{dJ}{dt} - \frac{4J}{3cp}\frac{d\rho}{dt} + \nabla \mathbf{H} = \kappa(S - J) \tag{9}$$

J is defined as a specific intensity averaged over all solid angles:

$$J(\mathbf{r}, t; \nu) = \frac{1}{4\pi}\int I(\mathbf{r}, t; \mathbf{n}, \nu) d\Omega \tag{10}$$

$$I = \frac{h^4\nu^3}{c^2} f_R \qquad d\Omega = \sin\theta d\theta d\varphi = -d\mu d\varphi \tag{11}$$

where f_R is the photon distribution function; that is $f_R(\mathbf{r}, t, \mathbf{n}, \mathbf{p})\, d\mathbf{p}$ is the number of photons, per volume, at place $\mathbf{r}$ and time t, with the momentum lying in the interval $(\mathbf{p}, \mathbf{p}+d\mathbf{p}) = (h\nu/c)(\mathbf{n}, \mathbf{n} + d\mathbf{n})$. The $\mathbf{n}$ value points into the direction of the propagation of the radiation. $\mathbf{H}$ is the Eddington flux, which is defined by the first angular momentum of the specific intensity

$$\mathbf{H}(\mathbf{r}, t; \nu) = \frac{1}{4\pi}\int I(\mathbf{r}, t; \mathbf{n}, \nu)\mathbf{n}\cos\vartheta d\Omega \qquad \vartheta = \angle(\mathbf{n}, \mathbf{r}) \tag{12}$$

This means that (8) and (9) describe isotropic media such as the spherical and the one-dimensional planar ones; ν designates the photon frequency, h is the Planck constant, and c is the speed of the light. For the sake of simplicity, we do not distinguish between the flux-mean opacity (in (8)) and the intensity-mean opacities (equations (3) and (9)). Such an approximation was also introduced by *Bogdan and Knölker* [1989] for the magnetized nonstratified solar atmosphere. The system of equations (1)–(3), (8), and (9) forms a full system of equations for the three scalar parameters ρ, p, T, and the vector $\mathbf{u}$.

3. Derivation of the Dispersion Equation

It is assumed that the plasma is gray and that it is slightly disturbed from its altitude-dependent equilibrium state (designated by the index "o")

$$\rho = \rho_o(z) \qquad \mathbf{u} = 0 \qquad p = p_o(z) \qquad T = T_o(z) \tag{13}$$

$$J(z) = J_o = \sigma T_o^4(z)/\pi \quad \mathbf{H} = 0 \tag{14}$$

The equilibrium value of the radiation intensity J_o is found equating I in (12) with the Kirchhoff-Planck function

$$B_\nu(T) = \frac{2h\nu^3}{c^2[\exp\{h\nu/k_B T\} - 1]} \tag{15}$$

and then averaging the result over the frequency ν. The equilibrium values of the source function S_o and the radiation intensity J_o coincide.

The plasma and field parameters may be expressed by the sum of the equilibrium values and the disturbances, which are designated by the index "1,"

$$\rho(\mathbf{r},t) = \rho_o(z) + \rho_1(\mathbf{r},t) \qquad \mathbf{u}(\mathbf{r},t) = \mathbf{v}(\mathbf{r},t) \tag{16}$$

$$p(\mathbf{r},t) = p_o(z) + p_1(\mathbf{r},t) \qquad T(\mathbf{r},t) = T_o(z) + T_1(\mathbf{r},t) \tag{17}$$

$$J(\mathbf{r},t) = \sigma T_o^4(z)/\pi + J_1(\mathbf{r},t) \qquad \mathbf{H}(\mathbf{r},t) = \mathbf{H}_1(\mathbf{r},t) \tag{18}$$

Further, an attempt is made to derive the dispersion equation of nonadiabatic waves in the nonhomogeneous solar substance and to find a way to obtain relations between the wave amplitudes of different plasma parameters. For the interpretation of experimental data on the solar atmosphere, it is especially important to study temperature and radiation intensity fluctuations. The method used here is analogous to the method applied for the case when the loss term $L = 0$ by *Roberts* [1991].

Taking into account the radiation pressure (6), the equilibrium momentum balance (2) is written as ($\mathbf{g} = -g\mathbf{n}_z$)

$$-\frac{\partial p_o}{\partial z} - \frac{4\pi}{3c}\frac{\partial J_o}{\partial z} = g\rho_o(z) \tag{19}$$

Further, expressing the mean radiation intensity J_o by (14) and $T_o(z)$ by the effective solar temperature

$$T_o^4(z) = T_{\text{eff}}^4\left[\frac{3}{4}\tau + \frac{1}{2}\right] \tag{20}$$

$$T_{\text{eff}}^4 = \frac{L_S}{4\pi\sigma r_S^2} \quad T_{\text{eff}} = 5777 \pm 2.5 \text{ K} \tag{21}$$

T_{eff} is the effective temperature of a blackbody of solar radius $r_S = (6.9626 \pm 0.0007) \times 10^8$ m, which radiates with the total solar luminosity $L_S = (3.845 \pm 0.006) \times 10^{26}$ W. Value τ is the optical depth of the solar atmosphere, which is assumed to be gray, which that means

$$4T_o^3\frac{\partial T_o}{\partial z} = \frac{3L_S}{16\pi\sigma r_S^2}\kappa_o(z) \tag{22}$$

and

$$\frac{\partial J_o}{\partial z} = \frac{3c}{4\pi}A\kappa_o(z) \quad A = \frac{L_S}{4c\pi r_S^2} = \text{const} \tag{23}$$

Finally, one obtains the equilibrium condition

$$\frac{\partial p_o}{\partial z} = -A\kappa_o(z) - g\rho_o(z) \tag{24}$$

where $A \approx 0.2105$ N m^{-2} is known. For g the gravitational acceleration at the solar surface $g = 274$ m s^{-2} will be used.

On the other side, T_o in the equilibrium formula $J_o = \sigma T_o^4/\pi$, can be expressed by the equation of state,

$$J_o = \frac{\sigma\tilde{\mu}^4 p_o^4}{\pi R^4 \rho_o^4} \tag{25}$$

Forming the z derivative of J_o given by (25), and equating the result with $\partial J_o/\partial z$ of (24), one has

$$\kappa_o = \frac{16\sigma}{3Ac}T_o^3\frac{\partial T_o}{\partial z} = -\frac{[g\rho_o + (p_o/\rho_o)\rho_o^*]}{A + \rho_o^4/(Cp_o^3)} \quad \rho_o^* = \frac{\partial \rho_o}{\partial z} \tag{26}$$

$$C = \frac{16\sigma\tilde{\mu}^4}{3AcR^4} = \frac{64\sigma\tilde{\mu}^4\pi r_S^2}{3R^4 L_s} \tag{27}$$

This relation gives a possibility to estimate κ_o using tables of ρ_o and p_o found within the standard solar model. A comparison of the results with available opacity values allows an evaluation of the quality of the theory developed here of nonadiabatic solar waves. It should be also mentioned that deriving (26) and (27), the mean molecular weight $\tilde{\mu}$ was taken to be altitude-independent. An improvement of (26) and (27) for altitude-dependent $\tilde{\mu}(z)$ can easily be done.

In the case of low-temperature solar plasma with log T < 3.85, opacity tables with an accuracy of about 10^{-3} are available (see, for example, the review by *Gong and Däppen* [1998]). In the solar atmosphere about 300 km thick ($\tau \lesssim 1$), the opacity averaged over the frequency is about 3×10^{-8} cm^{-1}. For the Sun in general, typical values of the opacity per unit of mass are 0.5×10^4 cm^2 g^{-1} [*Weigert and Wendker*, 1996].

Parameters of the photosphere and lower chromosphere of the Sun are presented in Table 1 and Figures 1 and 2 after *Weigert and Wendker* [1996]. On the basis of these parameters, opacities are calculated using the dependencies on T_o and $p_o - \rho_o$ given by (26). The results for the opacities are compared with the continuum opacity vales at 500 nm presented by *Kurucz* [1979] for solar regions with $T_o \lesssim 11{,}000$ K. It is clear that within the very simple model presented here with constant $\tilde{\mu}$, only the order of magnitude of the opacity in certain solar regions may be reproduced.

Considering the radiation equilibrium described by (8) and (9), one finds the relations

$$\frac{1}{3}\frac{\partial J_o}{\partial z} = -\kappa_o H_{oz} \tag{28}$$

and

$$\text{div}\mathbf{H}_o = 0 \tag{29}$$

Equation (28) shows that in the case of nonuniform temperature the mean Eddington flux does not vanish. Considering only vertical gradients in the plasma, relation (29) results into a constant z component of the mean Eddington flux. Expressing $\partial J_o/\partial z$ by (23), one obtains

$$H_{oz} = -\frac{Ac}{4\pi} = \text{const} \tag{30}$$

For the linearized momentum balance, one finds from (2)

$$\rho_o\omega v_x = k_x p_1 + \frac{4\pi}{3c}k_x J_1 \tag{31}$$

$$v_y = 0 \tag{32}$$

$$i\rho_o\omega v_z = \frac{\partial p_1}{\partial z} + \frac{4\pi}{3c}J_1^* + g\rho_1 \quad J_1^* = \frac{\partial J_1}{\partial z} \tag{33}$$

The linearized continuity equation has the form

$$-i\omega\rho_1 + \rho_o D^o + \mathbf{v}\ \text{grad}\rho_o = 0 \tag{34}$$

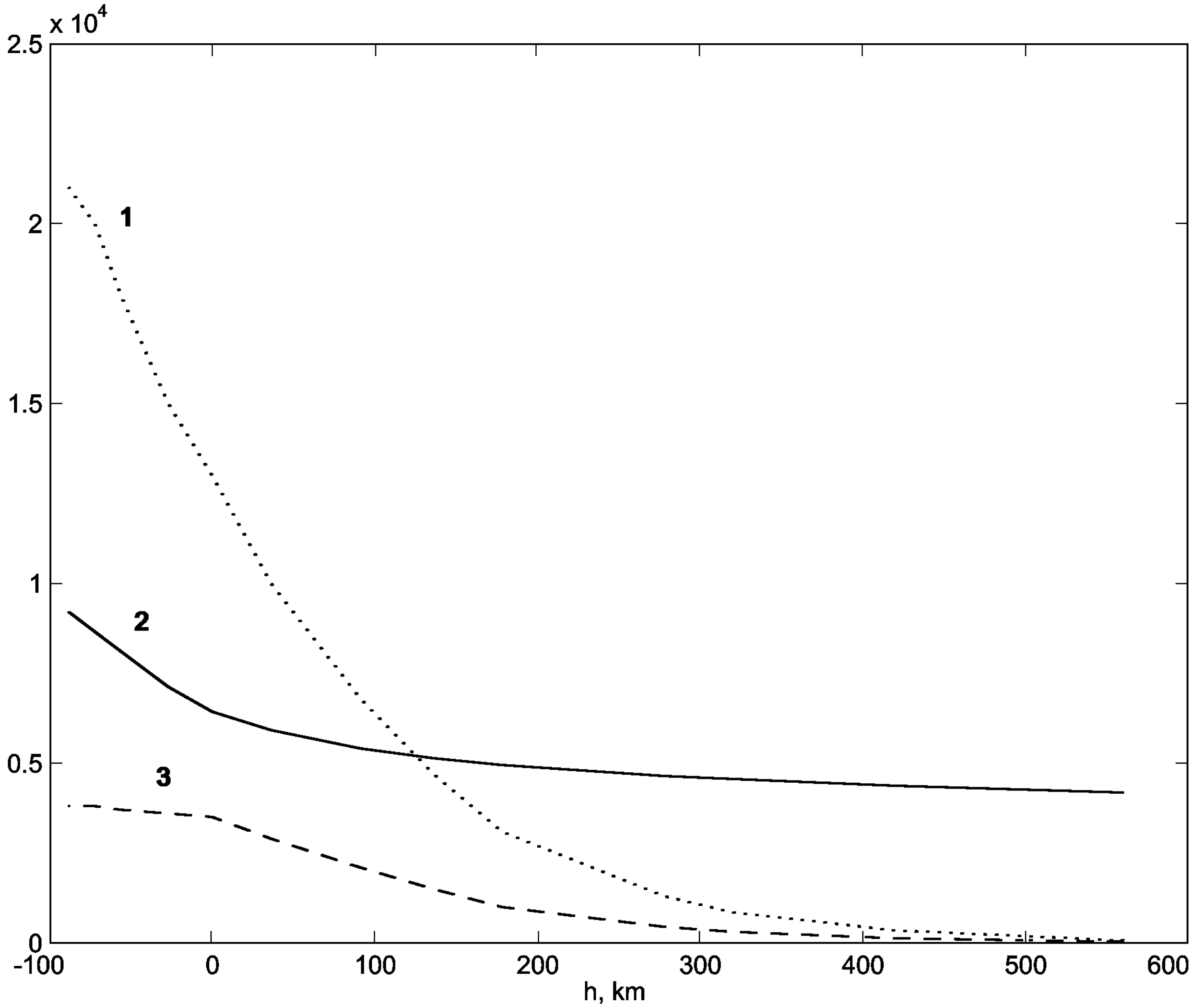

Figure 1. Parameters of the solar atmosphere (presented in Table 1) as a function of the altitude h. 1, the pressure p in Pa; 2, the temperature T in K; 3, the mass density ρ in 10^{-10} g cm^{-3}.

Here

$$D^o = \text{div } \mathbf{v} = ik_x v_x + \frac{\partial v_z}{\partial z} \tag{35}$$

The corresponding energy balance for the disturbances is (using $d\rho/dt = -\rho\nabla\mathbf{v}$)

$$-i\omega p_1 + v_z \frac{\partial p_o}{\partial z} + c_s^2 \rho_o D^o$$
$$= 4\pi(\gamma - 1)\kappa_o \left(J_1 - \frac{4\sigma T_o^3}{\pi} T_1 \right) \tag{36}$$

Expressing $\partial p_o/\partial z$ in (36) by (3), and T_1 by the linearized equation of state

$$\frac{T_1}{T_o} = \frac{p_1}{p_o} - \frac{\rho_1}{\rho_o} \tag{37}$$

one obtains

$$-i\omega p_1 - g\rho_o v_z - A\kappa_o v_z + c_s^2 \rho_o D^o$$
$$= 4\pi(\gamma - 1)\kappa_o \left(J_1 - \frac{4\sigma T_o^4}{\pi} \frac{p_1}{p_o} + \frac{4\sigma T_o^4}{\pi} \frac{\rho_1}{\rho_o} \right) \tag{38}$$

The equations (31)–(34) and (38) describe the disturbances of the plasma parameters ρ_1, $\mathbf{v}$, and p_1, which are coupled to the radiation field by J_1. The latter value has to be found from (8) and (9).

For the disturbances of the radiation field, one finds from (8), (9), and (30)

$$-\frac{i\omega}{c} H_{1x} + \frac{ik}{3} J_1 = -\kappa_o H_{1x} \tag{39}$$

$$-\frac{i\omega}{c} H_{1z} + \frac{1}{3} \frac{\partial J_1}{\partial z} = -\kappa_o H_{1z} \tag{40}$$

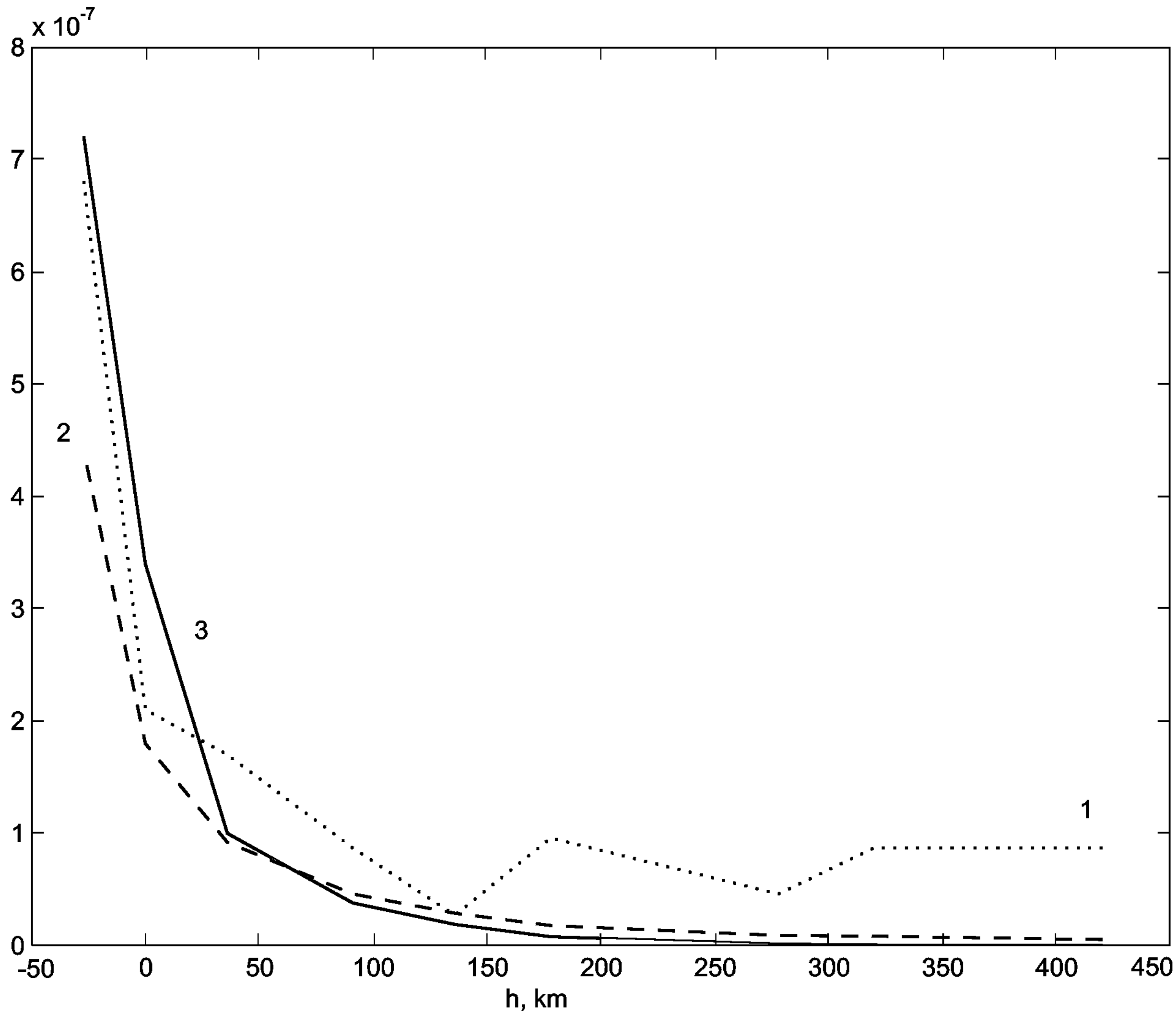

Figure 2. Absolute value of the opacity of the solar atmosphere (presented in Table 1) as a function of altitude h. 1, $\kappa_{p,\rho}$; 2, κ_T; 3, κ_K. The opacity is in cm^{-1} units.

$$
-\frac{i\omega}{c}J_1+\frac{v_z}{c}\frac{\partial J_o}{\partial z}+\frac{4J_o\rho_o}{3cp_o}D^o+ikH_{1x}+\frac{\partial H_{1z}}{\partial z}
=\kappa_o\left(\frac{4\sigma T_o^4}{\pi}\frac{p_1}{p_o}-\frac{4\sigma T_o^4}{\pi}\frac{\rho_1}{\rho_o}-J_1\right) \tag{41}
$$

Expressing $\partial J_o/\partial z$ in (41) by (23) and (28), and substituting H_{1x} by (39) and H_{1z} by (40), one obtains for the dependence of the radiation intensity J_1 on the plasma disturbances the following formula:

$$
-\frac{i\omega}{c}J_1+\frac{3A\kappa_o}{4\pi}v_z+\frac{4J_o\rho_o}{3cp_o}D^o+\frac{k^2}{3(\kappa_o-i\omega/c)}J_1
-\frac{J_1^{**}}{3(\kappa_o-i\omega/c)}+\frac{J_1^*\kappa_o^*}{3(\kappa_o-i\omega/c)^2}
=\kappa_o\left(\frac{4\sigma T_o^4}{\pi}\frac{p_1}{p_o}-\frac{4\sigma T_o^4}{\pi}\frac{\rho_1}{\rho_o}-J_1\right) \tag{42}
$$

$$
J_1^{**}=\frac{\partial^2 J_1}{\partial z^2}\qquad \kappa_o^*=\frac{\partial\kappa_o}{\partial z}
$$

Now (31), (33)–(35), (38), and (42) have to be used to derive the dispersion equation of the nonadiabatic waves. These equations describe the disturbances v_x, v_z, p_1, J_1, ρ_1, and D_o.

As a next step, ρ_1 will be excluded from the system of equations. Therefore ρ_1 from (34),

$$
\rho_1=\frac{v_z\rho^*+\rho_oD^o}{i\omega} \tag{43}
$$

is substituted into (33), (38), (42). One finds then

$$
\frac{\partial p_1}{\partial z}+\frac{4\pi}{3c}J_1^*+v_z\left[\frac{g\rho^*}{i\omega}-i\omega\rho_o\right]+\frac{g\rho_oD^o}{i\omega}=0 \tag{44}
$$

Table 1. Parameters of the Photosphere ($h \lesssim$300 km) and Lower Chromosphere ($h \gtrsim$300 km) of the Sun [*Weigert and Wendker*, 1996]

	h, km	τ	T, K	p, Pa	ρ, g cm^{-3}	$\kappa_{p,\rho}$, cm^{-1}	κ_T, cm^{-1}	$\log \kappa_K^*$	κ_K, cm^{-1}
1	560	10^{-4}	4180	71	3.6×10^{-9}	-1.5×10^{-7}	1.4×10^{-8}	-2.22	2.2×10^{-11}
2	420	10^{-3}	4370	350	1.3×10^{-8}	-8.7×10^{-8}	-5.4×10^{-9}	-2.10	1.0×10^{-10}
3	320	0.005	4560	850	3.1×10^{-8}	-8.7×10^{-8}	-8.6×10^{-9}	-1.68	6.5×10^{-10}
4	278	0.01	4640	1300	4.5×10^{-8}	-4.6×10^{-8}	-9.1×10^{-9}	-1.5	1.4×10^{-9}
5	178	0.05	4950	3100	1.0×10^{-7}	-9.6×10^{-8}	-1.8×10^{-8}	-1.1	7.9×10^{-9}
6	136	0.1	5140	4700	1.5×10^{-7}	-2.7×10^{-8}	-2.9×10^{-8}	-0.90	1.9×10^{-8}
7	91	0.2	5410	6800	2.1×10^{-7}	-8.7×10^{-8}	-4.6×10^{-8}	-0.74	3.8×10^{-8}
8	36	0.5	5920	10,000	2.9×10^{-7}	-1.7×10^{-7}	-9.2×10^{-8}	-0.46	1.0×10^{-7}
9	0	1.0	6430	13,000	3.5×10^{-7}	-2.1×10^{-7}	-1.8×10^{-7}	$+0.03$	3.8×10^{-7}
10	-27	2.0	7120	15,000	3.6×10^{-7}	-6.8×10^{-7}	-4.4×10^{-7}	$+0.3$	7.2×10^{-7}
11	-56	5.0	8100	18,000	3.7×10^{-7}	-9.7×10^{-7}	-8.6×10^{-7}	$+0.8$	2.3×10^{-6}
12	-72	10.0	8650	20,000	3.8×10^{-7}	-9.5×10^{-7}	-1.1×10^{-6}	$+1.08$	4.6×10^{-6}
13	-88	20.0	9200	21,000	3.8×10^{-7}	-1.7×10^{-6}	-1.3×10^{-6}	$+1.35$	8.5×10^{-6}

Value τ is the optical depth for the radiation at a wavelength of 500 nm, T is the temperature, p is the pressure, ρ is the density. $\kappa_{p,\rho}$ is the opacity calculated using the $p_o - \rho_o$ dependence given by (26), and κ_T designates the opacity calculated on the basis of the T_o dependence of (26). $\log \kappa_K^*$ is the log-value of the continuum mass absorption coefficient at 500 nm obtained by the convection model [*Kurucz*, 1979], and κ_K is the absolute value of the opacity found from κ_K^*. The altitude $h = 0$ is related to $\tau = 1$.

$$\left[\frac{\alpha\kappa_o T_o^4}{p_o} - i\omega\right] p_1 - \left[g\rho_o + A\kappa_o + \frac{\alpha\kappa_o T_o^4 \rho^*}{i\rho_o\omega}\right] v_z + \left[c_s^2\rho_o - \frac{\alpha\kappa_o T_o^4}{i\omega}\right] D^o = 4\pi(\gamma-1)\kappa_o J_1 \tag{45}$$

$$\left[\kappa_o - \frac{i\omega}{c} + \frac{k^2}{3(\kappa_o - i\omega/c)}\right] J_1 + \left[\frac{3A\kappa_o}{4\pi} + \frac{4\sigma T_o^4 \kappa_o \rho^*}{i\pi\rho_o\omega}\right] v_z + \left[\frac{4J_o\rho_o}{3cp_o} + \frac{4\sigma T_o^4\kappa_o}{i\pi\omega}\right] D^o - \frac{\kappa_o 4\sigma T_o^4}{\pi p_o} p_1 = \frac{J_1^{**}}{3(\kappa_o - i\omega/c)} - \frac{J_1^* \kappa_o^*}{3(\kappa_o - i\omega/c)^2} \tag{46}$$

$$\epsilon = \frac{4\pi}{3c} \qquad \alpha = 16(\gamma-1)\sigma \tag{47}$$

$$\beta = \frac{\rho_o c_s^3}{(\gamma-1)\sigma T_o^4} = \frac{16\gamma p_o c_s}{\alpha T_o^4} \tag{48}$$

Further, expressing v_x by (31), D^o one can transform (35) into

$$D^o = ikv_x + \frac{\partial v_z}{\partial z} = \frac{ik^2}{\rho_o\omega}(p_1 + \epsilon J_1) + \frac{\partial v_z}{\partial z} \tag{49}$$

and (44)–(46) and (49) describe p_1, J_1, v_z and D^o. Further, we try to exclude p_1 and v_z from the systems of equations, and to find an equation for D^o (according to the method of solution for the plasma without radiation) containing an additional J_1 contribution.

Thus first we exclude D^o from (44) to (46) and find p_1 and v_z as a function of J_1. Thus D^o from (49) will be substituted into (44)–(46) to obtain

$$\frac{gk^2}{\omega^2}(p_1 + \epsilon J_1) + v_z\left[\frac{g\rho^*}{i\omega} - i\omega\rho_o\right] + \frac{\partial p_1}{\partial z} + \epsilon J_1^* + \frac{g\rho_o}{i\omega}\frac{\partial v_z}{\partial z} = 0 \tag{50}$$

$$\left[\frac{\alpha\kappa_o T_o^4}{p_o} - i\omega + \frac{ik^2c_s^2}{\omega} - \frac{k^2\alpha\kappa_o T_o^4}{\rho_o\omega^2}\right] p_1 + \left[\frac{i\epsilon k^2 c_s^2}{\omega} - \frac{k^2\epsilon\alpha\kappa_o T_o^4}{\rho_o\omega^2} - 4\pi(\gamma-1)\kappa_o\right] J_1 - \left[g\rho_o + A\kappa_o + \frac{\alpha\kappa_o T_o^4\rho^*}{i\rho_o\omega}\right] v_z + \left[c_s^2\rho_o - \frac{\alpha\kappa_o T_o^4}{i\omega}\right]\frac{\partial v_z}{\partial z} = 0 \tag{51}$$

$$\left[-\frac{4\kappa_o\sigma T_o^4}{\pi p_o} + \frac{i4k^2 J_o}{3cp_o\omega} + \frac{4\sigma T_o^4\kappa_o k^2}{\pi\omega^2\rho_o}\right] p_1 + \left[\kappa_o - \frac{i\omega}{c} + \frac{k^2}{3(\kappa_o - i\omega/c)} + \frac{4ik^2\epsilon J_o}{3cp_o\omega} + \frac{4\sigma T_o^4\kappa_o k^2\epsilon}{\pi\omega^2\rho_o}\right] J_1 + \left[\frac{3A\kappa_o}{4\pi} + \frac{4\sigma T_o^4\kappa_o\rho^*}{i\pi\rho_o\omega}\right] v_z + \left[\frac{4J_o\rho_o}{3cp_o} + \frac{4\sigma T_o^4\kappa_o}{i\pi\omega}\right]\frac{\partial v_z}{\partial z} = \frac{J_1^{**}}{3(\kappa_o - i\omega/c)} - \frac{J_1^*\kappa_o^*}{3(\kappa_o - i\omega/c)^2} \tag{52}$$

Considering the limit of infinite Boltzmann number β (which is mathematically equivalent to $\sigma \to 0$, e.g., $J_o \to 0$)

in the energy balances (51) and (52), one can simplify the latter relations

$$\left[\frac{ik^2c_s^2}{\omega}-i\omega\right]p_1+\left[\frac{i\epsilon k^2c_s^2}{\omega}-4\pi(\gamma-1)\kappa_o\right]J_1$$

$$-\left[g\rho_o+A\kappa_o\right]v_z+c_s^2\rho_o\frac{\partial v_z}{\partial z}=0 \quad (53)$$

$$\left[\kappa_o-\frac{i\omega}{c}+\frac{k^2}{3(\kappa_o-i\omega/c)}\right]J_1+\frac{3A\kappa_o}{4\pi}v_z$$

$$=\frac{J_1^{**}}{3(\kappa_o-i\omega/c)}-\frac{J_1^*\kappa_o^*}{3(\kappa_o-i\omega/c)^2} \quad (54)$$

Substituting p_1 from (45) (in the limit $\sigma\to 0$)

$$-i\omega p_1-\left[g\rho_o+A\kappa_o\right]v_z+c_s^2\rho_oD^o=4\pi(\gamma-1)\kappa_oJ_1 \quad (55)$$

into (49) for D^o, one finds

$$\left(1-\frac{c_s^2k^2}{\omega^2}\right)D^o=\frac{J_1}{\rho_o\omega^2}\left[i\epsilon k^2\omega-4\pi(\gamma-1)\kappa_ok^2\right]$$

$$-\frac{v_z}{\rho_o\omega^2}\left[k^2g\rho_o+Ak^2\kappa_o\right]+\frac{\partial v_z}{\partial z} \quad (56)$$

Then, forming the z derivative of (55) and substituting into the result the expression for $\partial v_z/\partial z$ found from (56), one has

$$\frac{\partial p_1}{\partial z}=\frac{D^o}{i\omega}\left[c_s^2\rho_o^*+\rho_o(c_s^2)^*-\left(1-\frac{c_s^2k^2}{\omega^2}\right)(g\rho_o+A\kappa_o)\right]$$

$$+\frac{D^{o*}}{i\omega}c_s^2\rho_o-\frac{v_z}{i\omega}\left[g\rho_o^*+A\kappa_o^*+\left\{\frac{k^2g}{\omega^2}+\frac{A\kappa_ok^2}{\rho_o\omega^2}\right\}\right.$$

$$\left.\times(g\rho_o+A\kappa_o)\right]-4\pi(\gamma-1)\kappa_o^*J_1-4\pi(\gamma-1)\kappa_oJ_1^*$$

$$+\frac{k^2J_1}{i\rho_o\omega^3}(g\rho_o+A\kappa_o)[i\epsilon\omega-4\pi(\gamma-1)\kappa_o] \quad (57)$$

Now $\partial p_1/\partial z$ from (57) is substituted into (44). Using the relation

$$\frac{(c_s^2)^*}{c_s^2}=-\frac{\rho_o^*}{\rho_o}-\frac{\gamma g}{c_s^2}-\frac{A\gamma\kappa_o}{\rho_oc_s^2} \quad (58)$$

one finds

$$D^{o*}+D^o\left[\frac{k^2g}{\omega^2}-\frac{\gamma g}{c_s^2}-\frac{A\kappa_o}{\rho_oc_s^2}\left(1+\gamma+\frac{k^2c_s^2}{\omega^2}\right)\right]$$

$$+\frac{v_z}{c_s^2}G_1(\omega,k)+F_1(J_1,J_1^*)=0 \quad (59)$$

$$G_1(\omega,k)=\omega^2-\frac{g^2k^2}{\omega^2}-\frac{A\kappa_o^*}{\rho_o}-\frac{2Ak^2g\kappa_o}{\rho_o\omega^2}-\frac{A^2k^2\kappa_o^2}{\rho_o^2\omega^2} \quad (60)$$

$$F_1(J_1,J_1^*)=\frac{i\omega}{\rho_oc_s^2}\left[\epsilon J_1^*-4\pi(\gamma-1)\kappa_o^*J_1-4\pi(\gamma-1)\kappa_oJ_1^*\right]$$

$$+\frac{ik^2}{c_s^2\rho_o^2\omega}(g\rho_o+A\kappa_o)\left[\epsilon-\frac{4\pi(\gamma-1)\kappa_o}{i\omega}\right]J_1 \quad (61)$$

The z derivative of (59) is written as

$$D^{o**}+D^{o*}\left[\frac{k^2g}{\omega^2}-\frac{\gamma g}{c_s^2}-\frac{A\kappa_o}{\rho_oc_s^2}\left(1+\gamma+\frac{k^2c_s^2}{\omega^2}\right)\right]$$

$$+\frac{D_o}{c_s^2}\left(\frac{(c_s^2)^*}{c_s^2}\left(\gamma g+\frac{A\kappa_o}{\rho_o}+\frac{A\kappa_o\gamma}{\rho_o}\right)+\left[1+\gamma+\frac{k^2c_s^2}{\omega^2}\right]\right.$$

$$\left.\times\left(\frac{A\kappa_o\rho_o^*}{\rho_o^2}-\frac{A\kappa_o^*}{\rho_o}\right)+\left[1-\frac{k^2c_s^2}{\omega^2}\right]G_1(\omega,k)\right)$$

$$+\frac{v_z}{c_s^2}\left(\left(\frac{k^2g}{\omega^2}+\frac{A\kappa_ok^2}{\rho_o\omega^2}-\frac{(c_s^2)^*}{c_s^2}\right)G_1(\omega,k)+G_2(\omega,k)\right)$$

$$+F_2(J_1,J_1^*,J_1^{**})=0 \quad (62)$$

$$G_2(\omega,k)=\frac{A}{\omega^2\rho_o^2}\left[-\omega^2\rho_o\kappa_o^{**}+\omega^2\kappa_o^*\rho_o^*-2k^2g\kappa_o^*\rho_o\right.$$

$$\left.+2k^2g\kappa_o\rho_o^*-2Ak^2\kappa_o^*\kappa_o+\frac{2Ak^2\kappa_o^2\rho_o^*}{\rho_o}\right]$$

$$F_2(J_1,J_1^*,J_1^{**})=\frac{\partial F_1(J_1,J_1^*)}{\partial z}$$

$$+[4\pi(\gamma-1)\kappa_ok^2-i\epsilon k^2\omega]\frac{J_1G_1(\omega,k)}{\rho_o\omega^2c_s^2} \quad (63)$$

Multiplying (59) by

$$G_3(\omega,k)=\frac{k^2g}{\omega^2}+\frac{A\kappa_ok^2}{\rho_o\omega^2}-\frac{(c_s^2)^*}{c_s^2} \quad (64)$$

and subtracting (62) from the result, one obtains

$$D^{o**}+D^{o*}\left[\frac{(c_s^2)*}{c_s^2}-\frac{\gamma g}{c_s^2}-\frac{A\kappa_o}{\rho_oc_s^2}\left(1+\gamma+\frac{2k^2c_s^2}{\omega^2}\right)\right]$$

$$+D^o\left(\frac{\omega^2-k^2c_s^2}{c_s^2}+\frac{gk^2}{\omega^2}\left\{\frac{(c_s^2)^*}{c_s^2}+\frac{(\gamma-1)g}{c_s^2}\right\}\right.$$

$$+\frac{A\kappa_o\rho_o^*}{\rho^2}\left[\frac{2k^2}{\omega^2}+\frac{1+\gamma}{c_s^2}\right]-\frac{A\kappa_o^*(2+\gamma)}{\rho_oc_s^2}$$

$$\left.+\frac{A\kappa_ok^2}{\rho_o\omega^2}\times\left[\frac{3\gamma g}{c_s^2}+\frac{2k^2g}{\omega^2}+\frac{2A\kappa_o\gamma}{\rho_oc_s^2}+\frac{2A\kappa_ok^2}{\rho_o\omega^2}\right]\right)$$

$$+\frac{v_z}{c_s^2}G_2(\omega,k)-F_1(J_1,J_1^*)G_3(\omega,k)$$

$$+F_2(J_1,J_1^*,J_1^{**})=0 \quad (65)$$

In the case of a vanishing radiation transport $J=0$, $A=0$, (65) may be transformed into a simple relation:

$$\frac{d^2Q}{dz^2}+R^2Q=0 \qquad Q=\sqrt{\rho_o}c_s^2D^o \quad (66)$$

$$R^2 = \frac{\omega^2 - \omega_a^2}{c_s^2} + k^2 \left(\frac{\omega_g^2}{\omega^2} - 1 \right) \qquad (67)$$

$$\omega_a^2 = \frac{c_s^2 \rho^{*2}}{4\rho_o^2} \left[3 - 2\frac{\rho_o \rho_o^{**}}{\rho_o^{*2}} \right] \qquad \omega_g^2 = -\frac{g^2}{c_s^2} - \frac{g\rho_o^*}{\rho_o} \qquad (68)$$

where ω_a and ω_g are the acoustic cutoff frequency and the buoyancy frequency of a stratified atmosphere without radiation transport. For linear temperature profiles, a solution of (66) was first found by Lamb [e.g., *Roberts*, 1991]. This solution may be represented by confluent hypergeometric functions. In all other cases (66) and above all (65) for systems with radiation transport have to be solved numerically.

1. Therefore first the fluctuations of the intensity of the scattered radiation field have to be found. Thus D_o from (56) (with v_z from (54) and $\partial v_z/\partial z$ from the z derivative of (54)) and D^{o*} from the z derivative of (56) (with v_z, $\partial v_z/\partial z$, $\partial^2 v_z/\partial z^2$ from (54) and its z derivatives) have to be substituted into (59). Consequently, a differential equation of the fourth order for J_1 with respect to z will be found, which has to be solved for given boundary conditions.

2. If one has obtained J_1, the solution for v_z may be obtained from (54).

3. Knowing J_1 and v_z, one may further determine p_1 using, for example, (53).

4. After that, it is possible to find v_x using (31).

5. Then, with the help of (42) (expressing D^o by (56)), ρ_1 may also be found.

6. From (37), one determines then temperature fluctuations especially important for interpretation of satellite experiments T_1.

7. Besides, from (39) and (40), one may obtain H_{1x} and H_{1z}, respectively.

Applications of this method to find the fluctuations of the parameters of the solar plasma and the radiation field will be presented in further papers.

3. Conclusions

In the present paper, former models of hydrodynamic adiabatic atmospheric waves are extended to the case of nonadiabatic acoustic-gravity waves in a system with radiation transport where the radiation relaxation is described in the Eddington approximation.

Effects of the stratification of the atmosphere are taken into account, but no influence of the mean magnetic field is considered. The temperature distribution is assumed to be nonuniform, and an altitudinal-dependent radiation pressure is taken into account in the momentum balances.

In analogy to the works for stratified plasmas neglecting nonadiabatic effects, a dispersion relation is found introducing the divergency of the plasma velocity as a new parameter. In the obtained dispersion relation, there appear new radiation-intensity–dependent coefficients and additional terms describing the wave damping by the radiation transport.

The momentum and energy balances for the particles and the radiation are studied here, contrary to other works. Thus a simple approximate relation between the opacity and the mean plasma pressure, the mean pressure gradient and the mass density is found. This relation may be used to estimate the quality of the model developed for the atmospheric acoustic-gravity waves. Since, for the sake of simplicity, the mean-molecular-weight gradients in the system are neglected here, one is able to estimate only the order of magnitude of the opacity.

A scheme is given, which indicates how to find successively the amplitudes of the plasma (velocity, mass density, pressure, temperature) and radiation field (radiation intensity, Eddington flux) parameters in a stratified atmosphere where hydrodynamic acoustic-gravity waves exist. To apply this scheme, boundary conditions for the radiation intensity fluctuations and their spatial derivaties up to the fourth order have to be taken into account.

Analytical expressions to calculate the amplitudes of the plasma and radiation field variations are presented for the case of a large Boltzmann number of the substance, i.e. for relatively cold atmospheres with high plasma pressure and rather low mass density, or for a substance with a polytropic coefficient of the order of unity. The obtained results may be used for the interpretation of experimental data obtained during the solar satellite experiments, e.g., CORONAS and SOHO.

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